Chapters 10 & 11: Rotational Dynamics
Thursday March 8th

- Review of rotational kinematics equations
- Review and more on rotational inertia
- Rolling motion as rotation and translation
- Rotational kinetic energy
- Rotational vectors
- Angular momentum (if time)
- Examples and demonstrations

- Note that there will be a Mini Exam on this material on the Thursday right after Spring Break

Reading: up to page 195 in Ch. 11

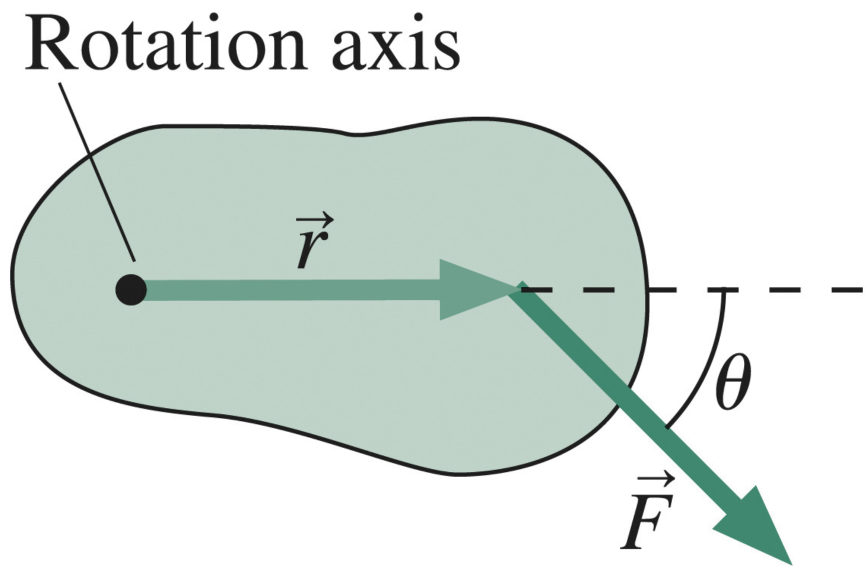
Review: Rotational Kinematic Equations

See previous class notes for definitions of rotational variables and transformations between linear and rotational variables.

Equation number	Equation	Missing quantity
10.7	$\omega = \omega_0 + \alpha t$	$\theta - \theta_0$
10.8	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$	ω
10.9	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	t
10.6	$\theta - \theta_0 = \bar{\omega} t = \frac{1}{2}(\omega_0 + \omega)t$	α
	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$	ω_0

Important: equations apply ONLY if angular acceleration is constant.

Review: Torque and Newton's 2nd Law



Definition: $$\tau = \left| \vec{r} \times \vec{F} \right| = rF \sin\theta$$

Newton's 2nd law: $$\tau = I\alpha$$

Rotational equivalent of force

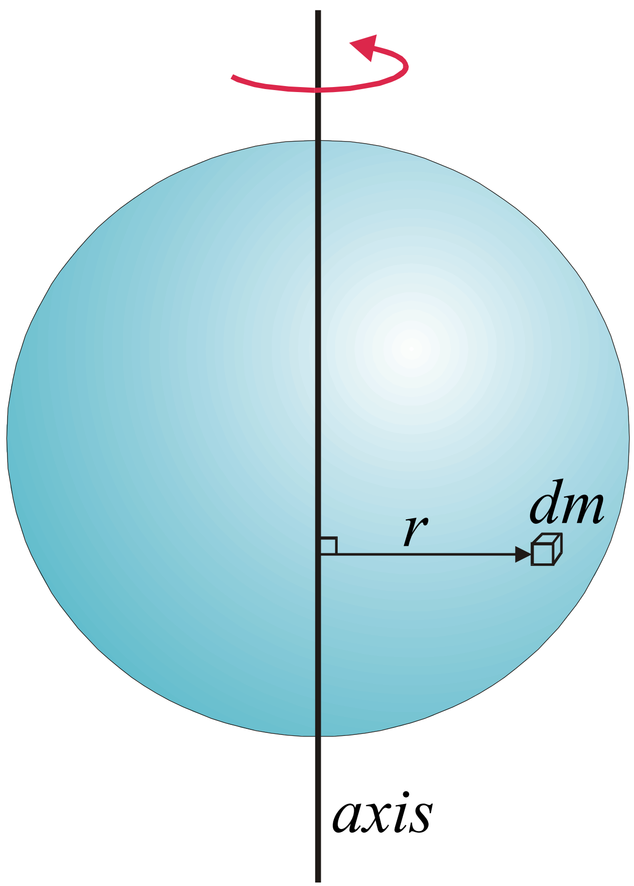
Rotational equivalent of mass

Rotational acceleration

Calculating Rotational Inertia

For a rigid system of discrete objects: $I = \sum m_i r_i^2$

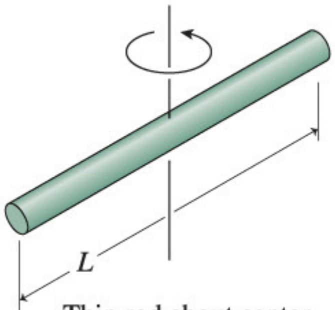
Therefore, for a continuous rigid object: $I = \int r^2 \, dm = \int \rho r^2 \, dV$



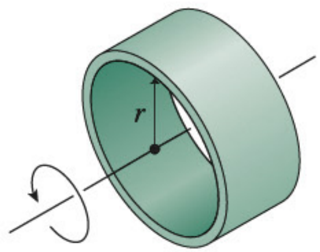
- Finding the moments of inertia for various shapes becomes an exercise in volume integration.
- You will not have to do such calculations.
- However, you will need to know how to calculate the moment of inertia of rigid systems of point masses.
- You will be given the moments of inertia for various shapes.

Rotational Inertia for Various Objects

Table 10.2 Rotational Inertias

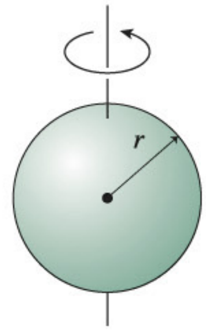


Thin rod about center
$I = \frac{1}{12}ML^2$

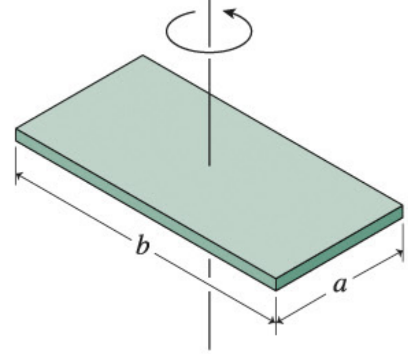


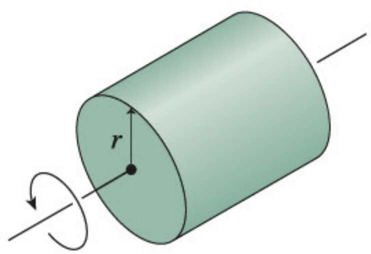
Thin ring or hollow cylinder about its axis
$I = MR^2$

Solid sphere about diameter
$I = \frac{2}{5}MR^2$



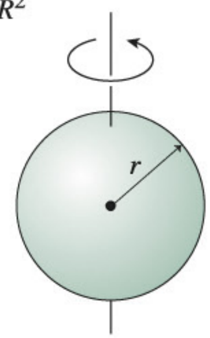
Flat plate about perpendicular axis
$I = \frac{1}{12}M(a^2 + b^2)$



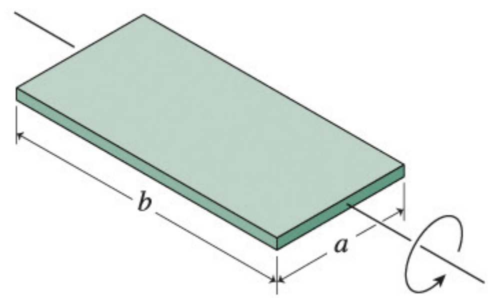


Disk or solid cylinder about its axis
$I = \frac{1}{2}MR^2$

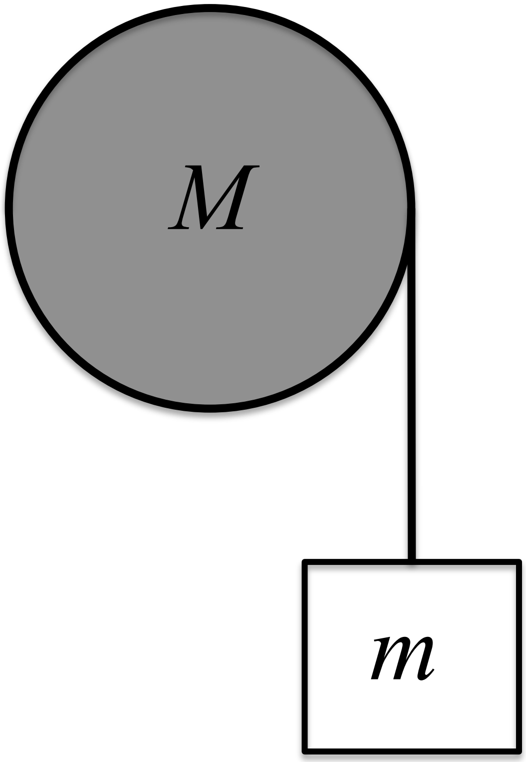
Hollow spherical shell about diameter
$I = \frac{2}{3}MR^2$



Flat plate about central axis
$I = \frac{1}{12}Ma^2$

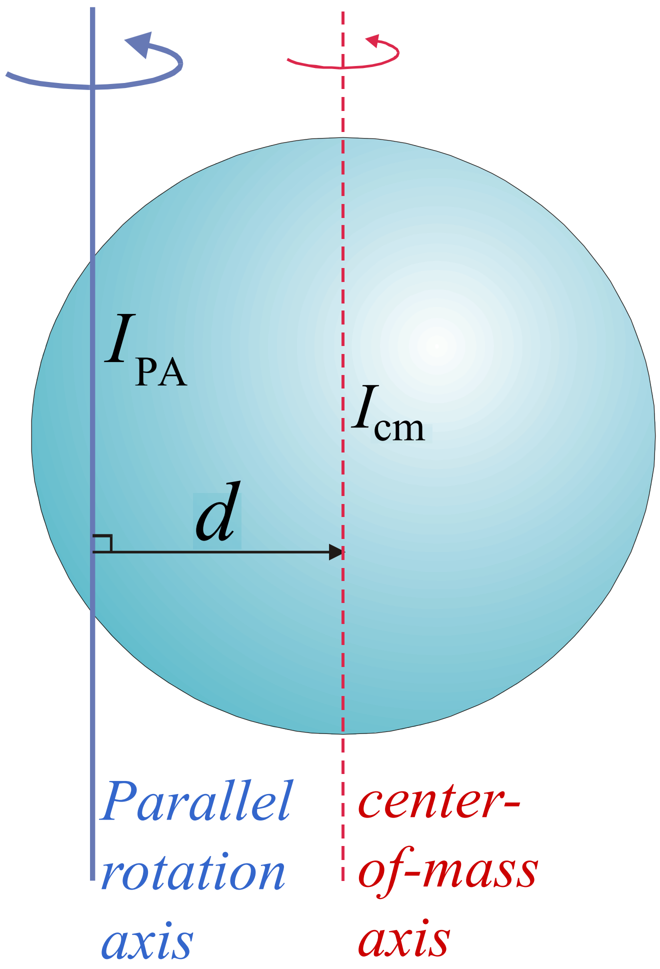


Example Problem



The pulley shown on the left can be treated as a uniform disk of radius 15 cm and mass $M = 470$ g. It is free to rotate without friction. Mass $m = 220$ g is attached to a light string and suspended over the pulley as shown. What is the resultant downward acceleration of mass m?

Parallel Axis Theorem



So I_{cm} is always the minimum value of I for a given object.

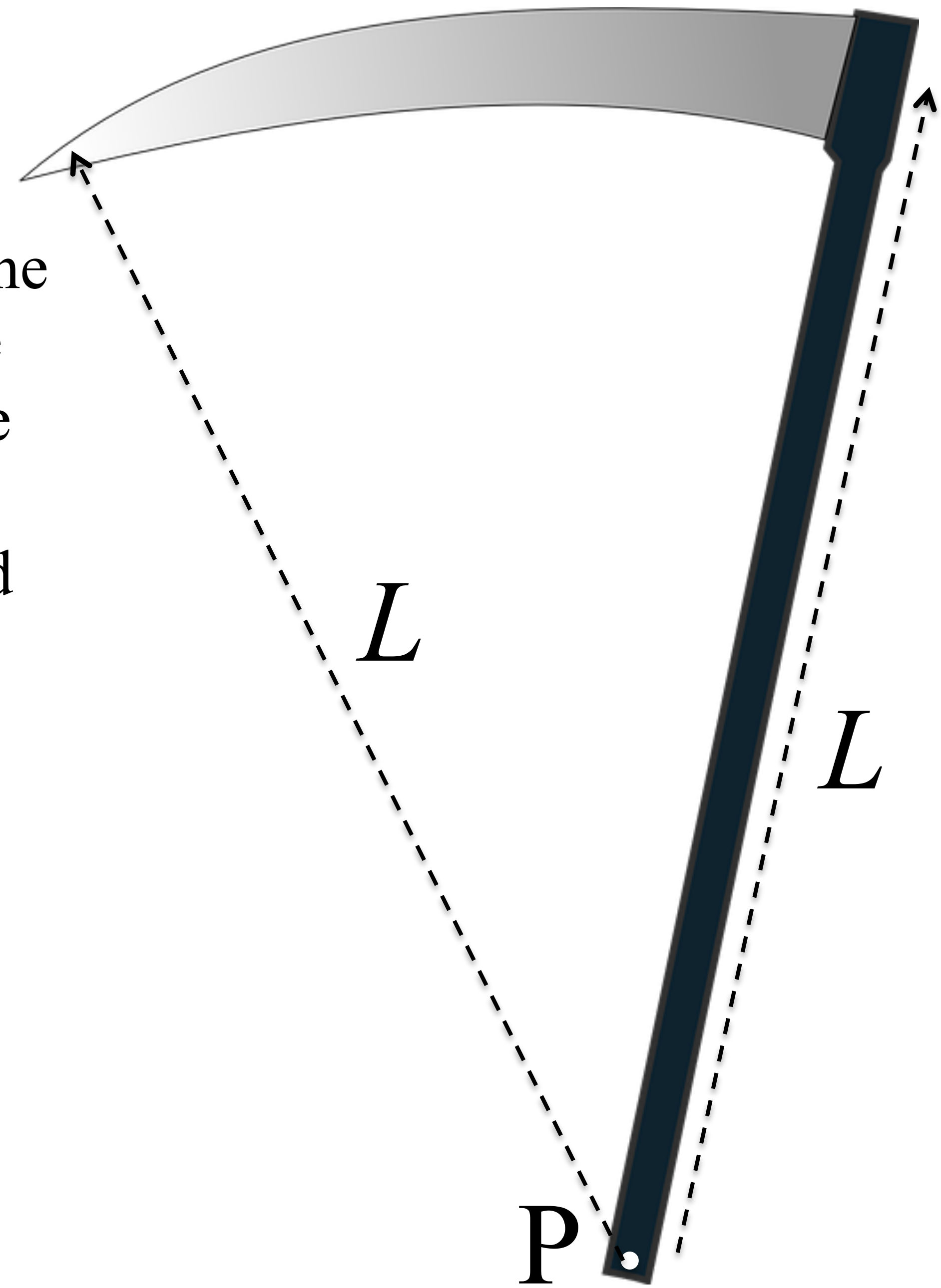
- If you know the moment of inertia of an object about an axis though its center-of-mass (cm), then it is trivial to calculate the moment of inertia of this object about any parallel axis:

$$I_{PA} = I_{cm} + Md^2$$

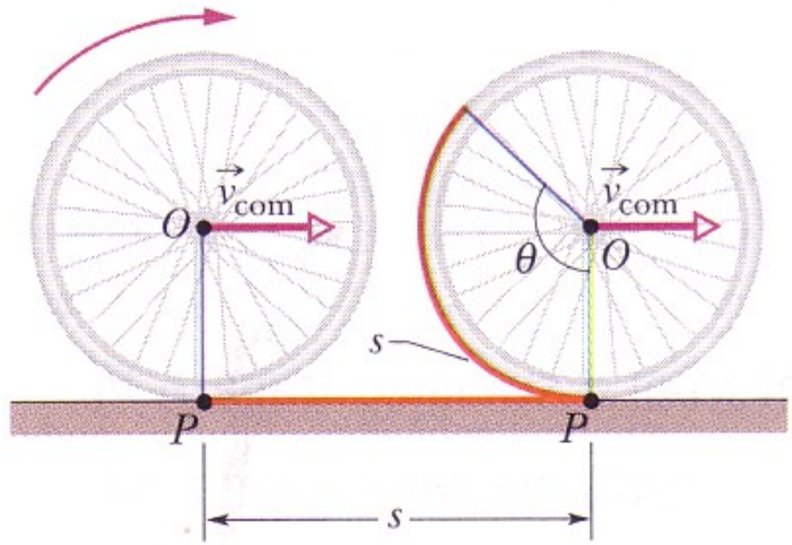
- Here, I_{cm} is the moment of inertia about an axis through the center-of-mass, and M is the total mass of the rigid object.
- It is essential that these axes are parallel; as you can see from table 10-2, the moments of inertia can be different for different axes.

Example Problem

Estimate the moment of inertia of the scythe shown on the right about the end of the handle (point P). Assume the blade has mass M, and the straight handle also has mass M and length L.



Rolling Motion as Rotation and Translation

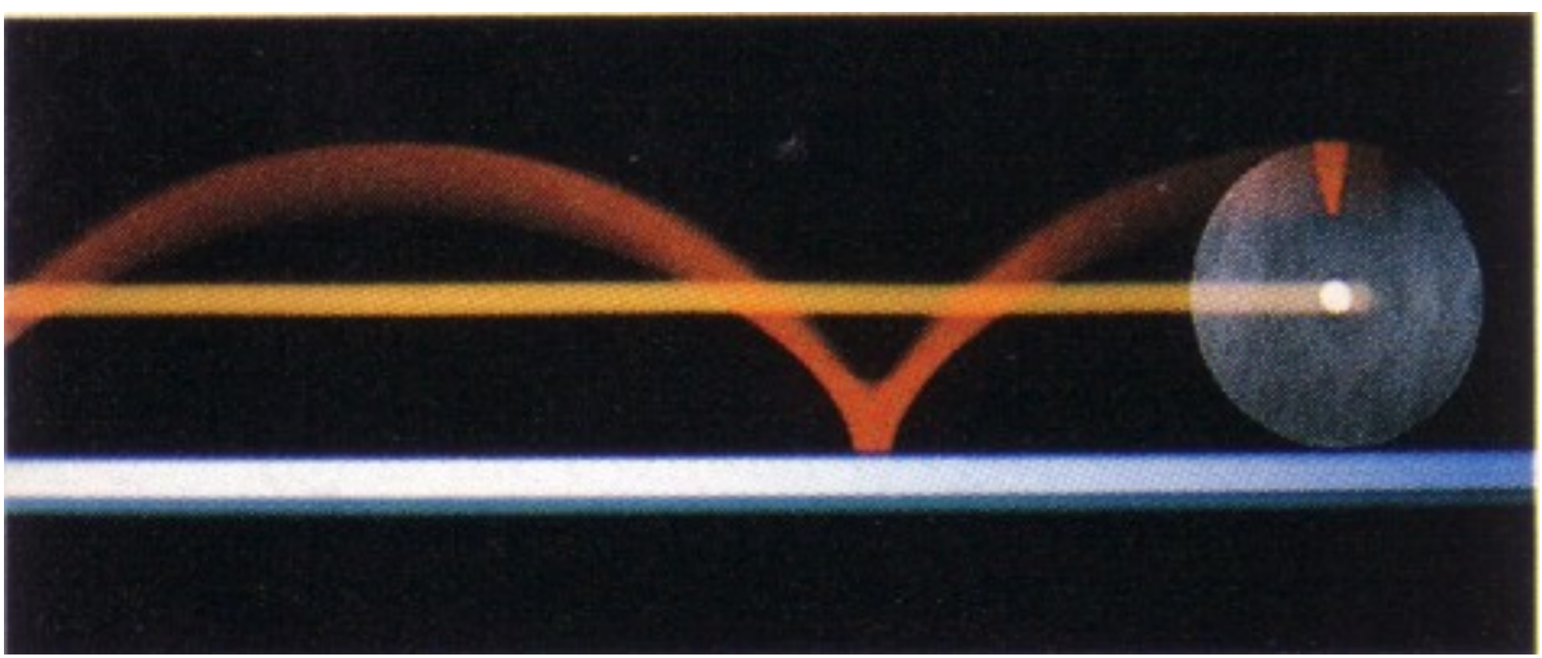


$$s = R\theta$$

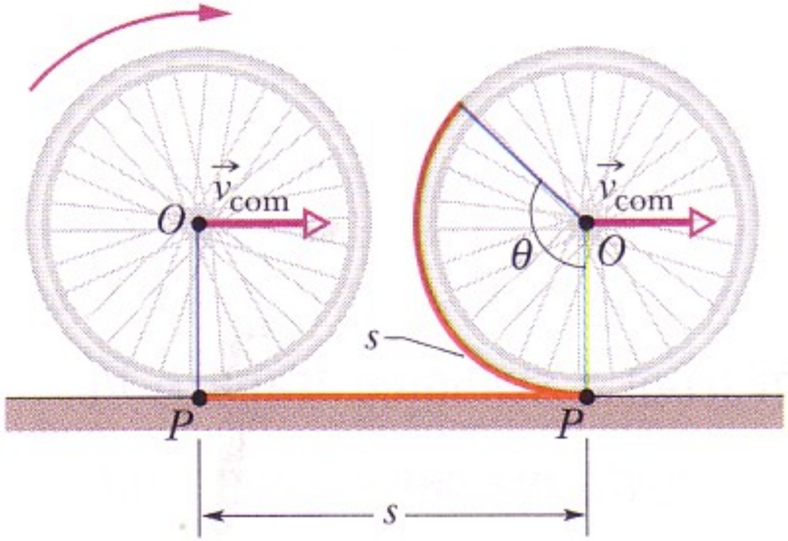
The wheel moves with speed ds/dt

$$\Rightarrow v_{cm} = R\omega$$

Another way to visualize the motion:



Rolling Motion as Rotation and Translation



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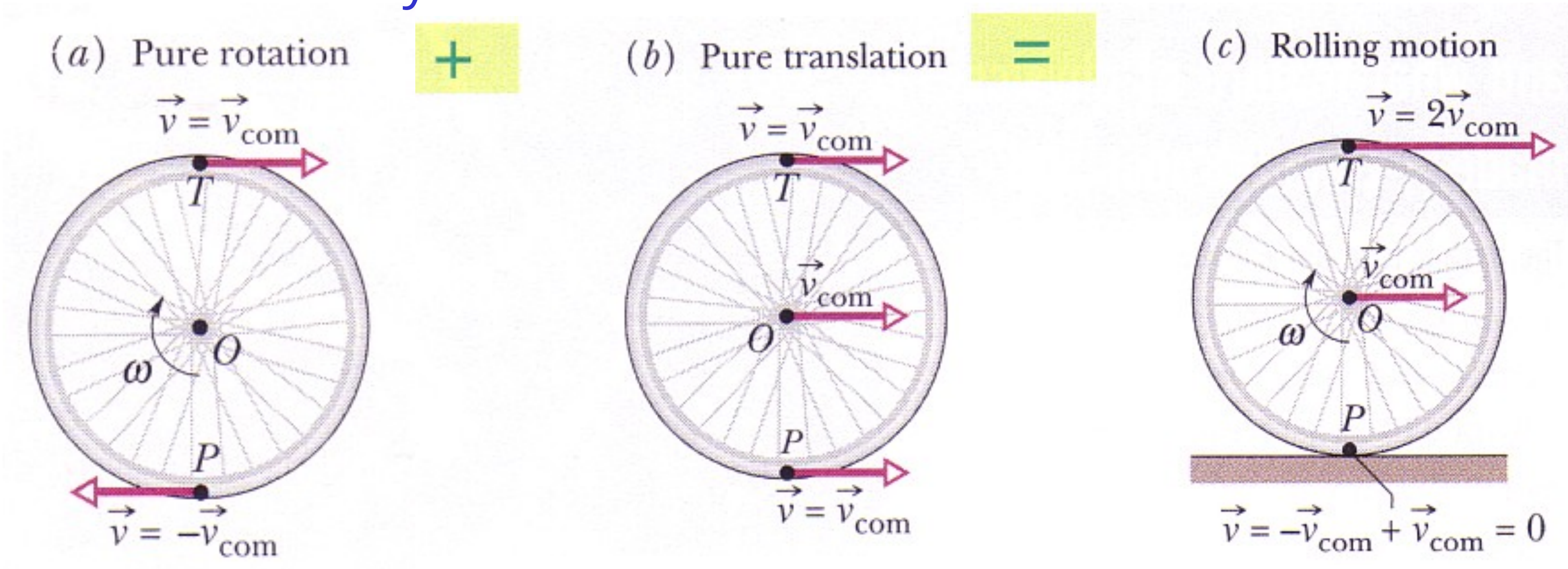
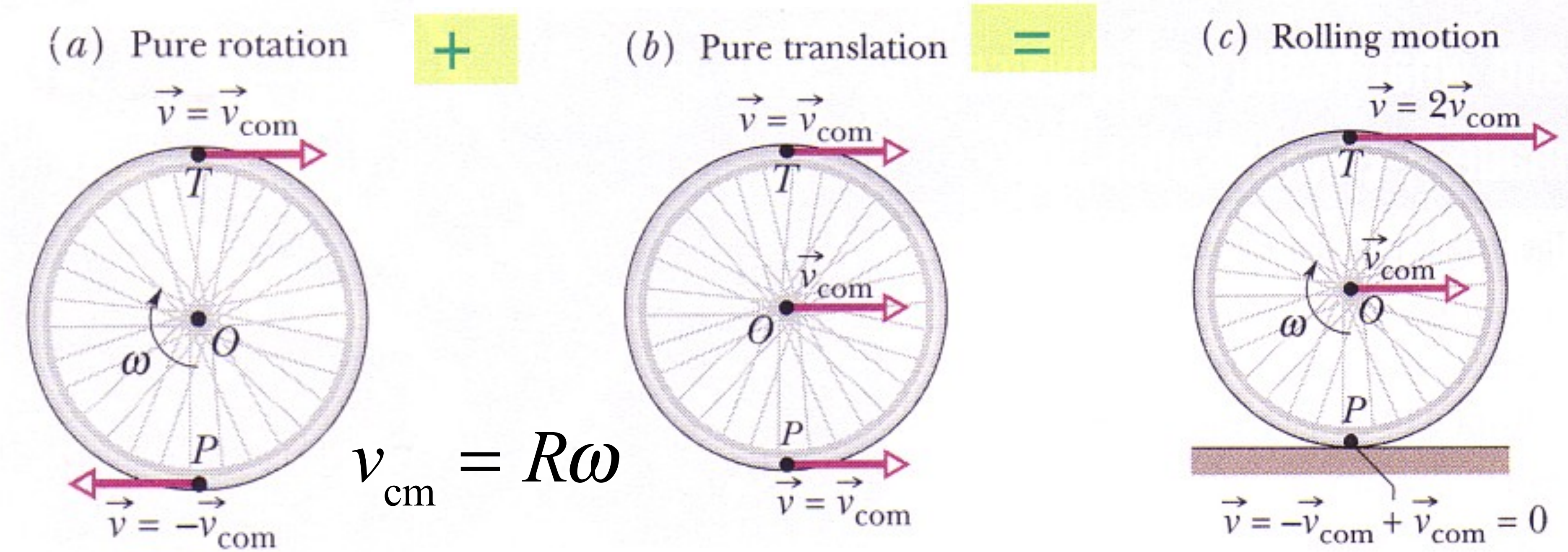


Rolling Motion as Rotation and Translation



$$v_{cm} = R\omega$$

Kinetic energy consists of rotational & translational terms:

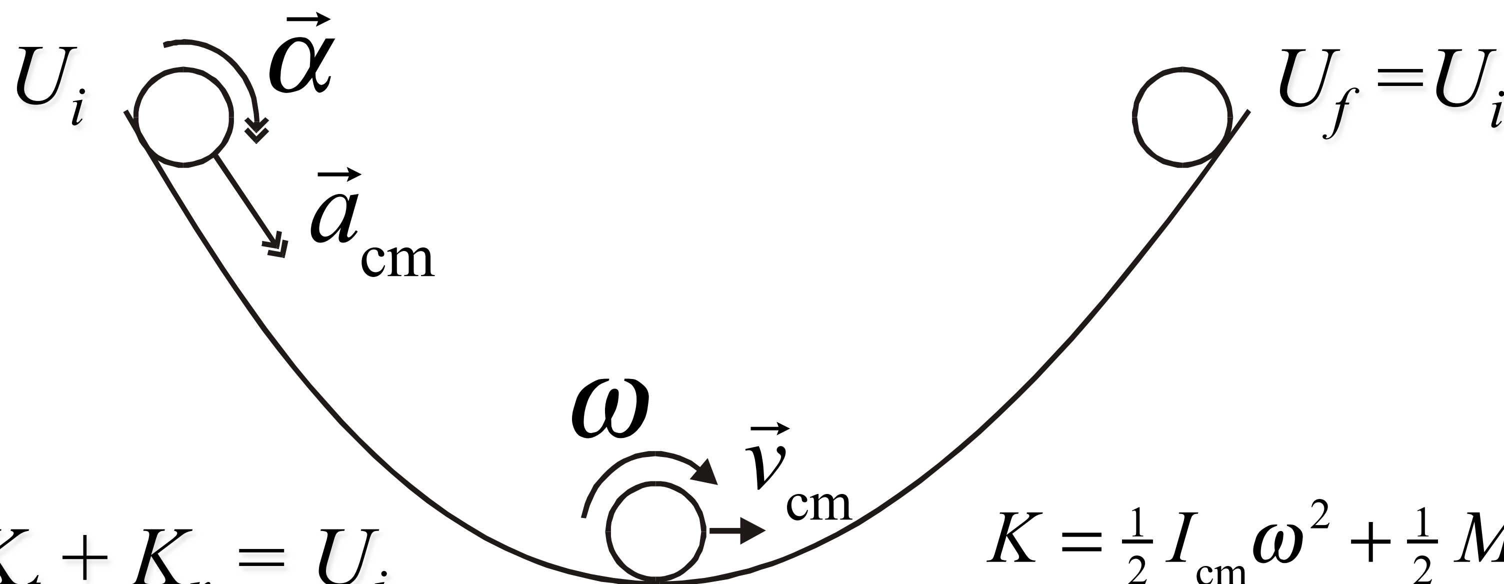
$$K = \tfrac{1}{2} I_{cm}\omega^2 + \tfrac{1}{2} M v_{cm}^2 = K_r + K_t$$

$$K = \tfrac{1}{2}\left\{ fMR^2 \right\}\frac{v_{cm}^2}{R^2} + \tfrac{1}{2} M v_{cm}^2 = \tfrac{1}{2} M v_{cm}^2 \times \left(1 + f\right)$$

Modified *K*: $K_{roll} = \left(1 + f\right) K_{trans}$ (look up *f* in Table 10.2)

Rolling Motion, Friction & Energy Conservation

- Friction plays a crucial role in rolling motion:
  - without friction a ball would simply slide without rotating;
  - Thus, friction is a necessary ingredient.
- However, if an object rolls without slipping, mechanical energy is **NOT** lost as a result of frictional forces, which do **NO** work.
  - An object must slide/skid for the friction to do work.
- Thus, if a ball rolls down a slope, the potential energy is converted to translational and rotational kinetic energy.



$$K = K_t + K_r = U_i$$

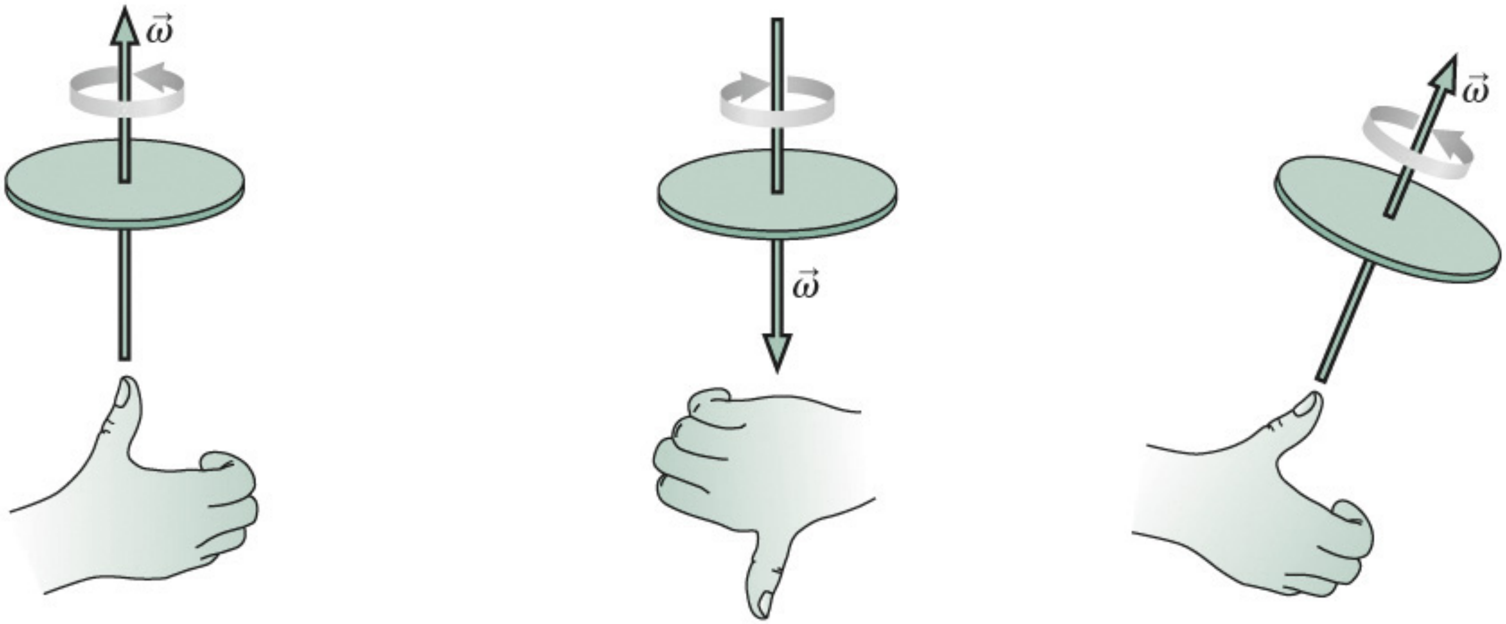
$$K = \tfrac{1}{2} I_{\text{cm}} \omega^2 + \tfrac{1}{2} M v_{\text{cm}}^2$$

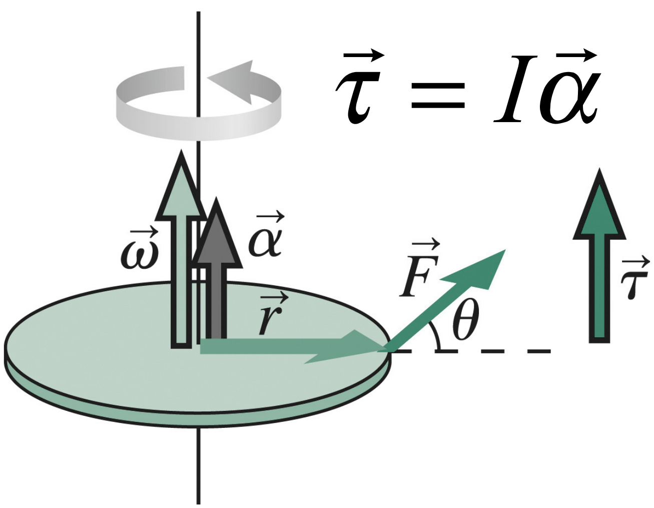
Example Problem

Three round objects, (i) a disk, (ii) a hoop, and (iii) a solid sphere, are released from the top of the ramp shown below. The objects have the same and mass, equal to 5 kg. The release point is 1 m above the horizontal section of the track. Calculate the resultant velocities of the three objects when they reach the bottom. You should assume that they roll without slipping.

Angular Quantities Have Direction

The direction of angular velocity is given by the **right-hand rule.**





$$\vec{\tau} = I\vec{\alpha}$$

Same applies to torque:

Torque is perpendicular to both the force vector and the displacement vector from the rotation axis to the force application point.

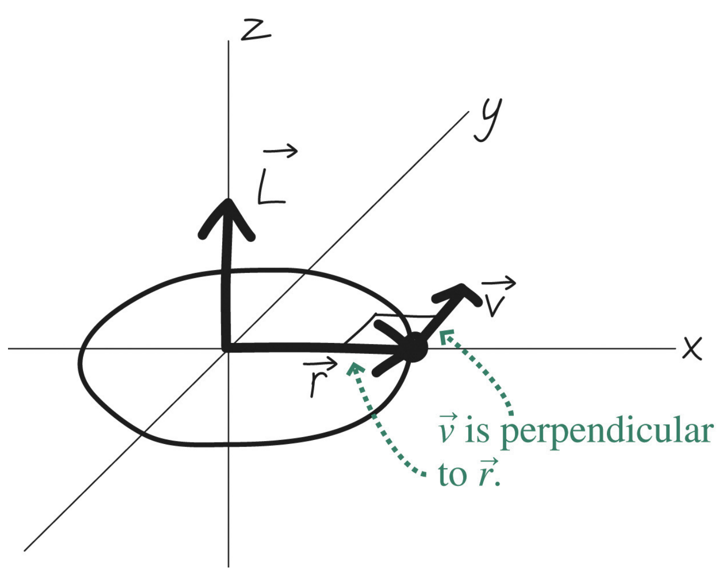
$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \left(\left|\vec{\tau}\right| = rF\sin\theta\right)$$

Review: Angular Momentum

- For a single particle, angular momentum is a vector given by the cross product of the displacement vector from the rotation axis with the linear momentum of the particle:

Angular momentum $\vec{L}$ is defined as: $\vec{L} = \vec{r} \times \vec{p} = m\left(\vec{r} \times \vec{v}\right)$

$$L = rp\sin\phi = mvr\sin\phi$$

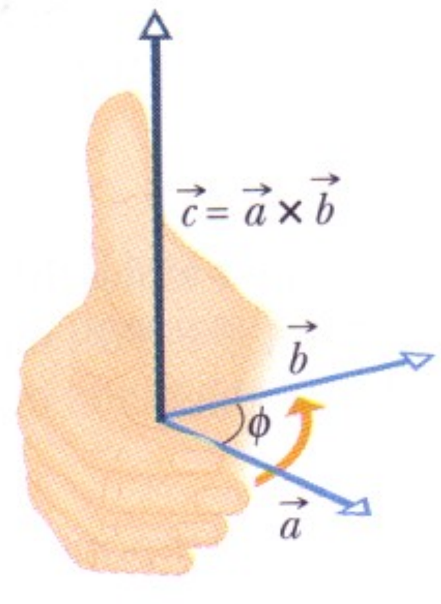


- For the case of a particle in a circular path, $L = mvr$, and is upward, perpendicular to the circle.
- For sufficiently symmetric objects, angular momentum is the product of rotational inertia (a scalar) and angular velocity (a vector):

$$\vec{L} = I\vec{\omega}$$

•SI unit is Kg.m^2/s.

The Vector Product, or Cross Product



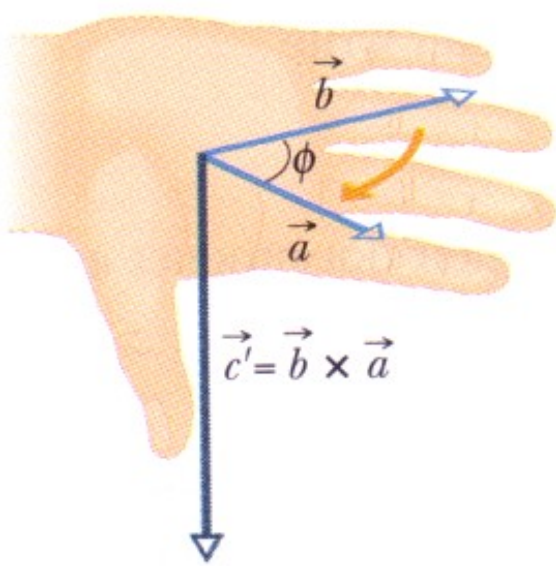
(a)

$$\vec{a} \times \vec{b} = \vec{c}, \text{ where } c = ab\sin\phi$$

$$\vec{a} \times \vec{b} = -\left(\vec{b} \times \vec{a}\right)$$

Direction of $\vec{c} \perp$ to both $\vec{a}$ and $\vec{b}$

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$



$$\hat{i} \times \hat{j} = \hat{k} \qquad \hat{j} \times \hat{i} = -\hat{k}$$

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$$\hat{k} \times \hat{i} = \hat{j} \qquad \hat{i} \times \hat{k} = -\hat{j}$$