1D Kinematics

Oscillator (01):

$$\frac{d}{dt}\,\sin(\omega\,t) = \omega\,\cos(\omega\,t)\,, \quad \frac{d}{dt}\,\cos(\omega\,t) = -\omega\,\sin(\omega\,t)\,.$$

Uniformly **increasing velocity (02)**. Acceleration:

$$a = \frac{v_2 - v_1}{t_2 - t_1}\,.$$

New t_0, t, initial v_0:

$$v_0 = v_1 + a\,(t_0 - t_1)\,, \quad x = v_0\,(t - t_0) + \frac{1}{2}\,a\,(t - t_0)^2\,.$$

Non-constant acceleration (04):

$$v(t) = \int dt\,a(t) + v_0\,, \quad x(t) = \int dt\,v(t) + x_0\,.$$

Free fall acceleration (06) A. Free fall with v_0 given:

$$x(t) = x_0 + v_0\,t - g\,\frac{t^2}{2}\,, \quad x = x_0 = 0 \quad \Rightarrow \quad 0 = v_0 - g\,\frac{t}{2} \quad \Rightarrow \quad t\,.$$

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Free fall acceleration B. Greatest height:

$$0 = \frac{dx}{dt} = v_0 - g\,t_{\max} \quad \Rightarrow \quad t_{\max} = \frac{v_0}{g}\,, \quad x_{\max} = x\big(t_{\max}\big)\,.$$

Free fall acceleration C. Initial velocity v_0, time at first half velocity, $v = v_0/2$:

$$v(t) = v_0 - g\,t\,, \quad \frac{v_0}{2} = v_0 - g\,t \quad \Rightarrow \quad t\,.$$

Free fall (07) from heights h and $h/2$:

$$h = v_0\,t + \frac{1}{2}\,g\,t^2\,, \quad \frac{h}{2} = \frac{1}{2}\,g\,t^2 \quad \Rightarrow \quad \frac{h}{2} = v_0\,t\,, \quad t = \sqrt{\frac{h}{g}} \quad \Rightarrow \quad v_0 =$$