Vectors and 2D Motion

Cartesian and polar coordinates (1).
Length of a 2D vector; angle ϕ:

$$r = \sqrt{x^2 + y^2}\,, \qquad \phi = \arctan(y/x)\,.$$

Components of a velocity:

$$v_x = v\,\cos(\phi)\,, \qquad v_y = v\,\sin(\phi)\,.$$

Not that CAPA wants asin, acos, atan and gives wrong when useing arc instead.

Adding vectors (2):

$$\vec{v}_3 = \vec{v} = \vec{v}_1 + \vec{v}_2$$

with $\vec{v}_1$ along the positive x-axis. Maximum and minimum magnitudes of $\vec{v}$:

$$v_{\rm max} = v_1 + v_2\,, v_{min} = v_1 - v_2\,.$$

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Inexperienced pilot (3).

Speed over ground v; speed in still air v_y:

$$v = \frac{r}{t}\,, \qquad v_y = \frac{y}{t}\,.$$

Let the speed in still air now v (our previous v_y). Direction angle θ:

$$v_x = v\,\sin(\theta) \;\Rightarrow\; \theta = arcsin(v_x/v)\,.$$

Ferry boat (4).

Acceleration, Velocity and Displacement Vector (5).

Velocity from acceleration $\vec{a}$ and initial velocity $\vec{v}_0$ at time $t = 0$:

$$\vec{v} = \vec{v}_0 + \vec{a}\,t\,, \quad v = \sqrt{v_x^2 + v_y^2}\,.$$

Angle with the convention: 0 to π for $v_y > 0$, 0 to $-\pi$ for $v_y < 0$. So,

$$\alpha = sign(v_y)\,\arccos(v_x/v)\,, \quad sign(v_y) = \pm 1\,.$$

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Position vector at time t with initial position $\vec{r}_0$:

$$\vec{r} = \vec{r}_0 + \vec{v}_0\, t + \frac{1}{2}\,\vec{a}\,t^2\,, \quad r = \sqrt{r_x^2 + r_y^2}\,.$$

Angle as before:

$$\alpha = sign(r_y)\,\arccos(r_x/r)\,, \quad sign(r_y) = \pm 1\,.$$

Velocity in the *xy* plane (6).

When particle is moving in x direction:

$$v_y = \frac{dr_y}{d\,t} = 0 \quad \Rightarrow \quad t \quad \Rightarrow \quad v_x = \frac{dr_x}{d\,t}\,.$$

When particle is moving in y direction:

$$v_x = \frac{dr_x}{d\,t} = 0 \quad \Rightarrow \quad t \quad \Rightarrow \quad v_y = \frac{dr_y}{d\,t}\,.$$