Oscillations II - 1

Oscillations (1): Consider

$$x(t) = A\cos(\omega\, t + \pi)$$

and its derivatives with respect to time t.

Energy and Amplitude in simple Harmonic Motion (2):

$$E_{\rm tot} = \frac{1}{2}\, M\, v_{\rm max}^2\,, \quad x_{\sf max} = \frac{v_{\sf max}}{\omega}\,.$$

Spring and Rolling Sphere (or Cylinder) (3). Energy:

$$U = \frac{1}{2}\, k\, x^2\,, \quad K = \frac{7}{10}\, M\, v^2 \quad \left(\text{or} \;\; K = \frac{3}{4}\, M\, v^2\right)\,.$$

Angular velocity:

$$0 = k\, x\, \dot{x} + \frac{7}{5}\, M\, \dot{x}\, \ddot{x} \quad \left(\text{or} \;\; \frac{3}{2}\, M\, \dot{x}\, \ddot{x}\right)\,.$$

Oscillations II - 2

Pendulum in an Accelerating Frame (4):

$$T \sim \sqrt{\frac{1}{g \pm a}}\,.$$

Pendulum in Rocket Ship (5):

$$L\,\ddot{\theta} = -m\,g\,\sin(\theta) \approx -m\,g\,\theta \;\Rightarrow\; \ddot{\theta} = -\omega^2\,\theta \text{ with } \omega = \sqrt{\frac{m\,g}{L}}\,,\; T = \frac{2\pi}{\omega}\,.$$

Pendulum (6). Equate the maximum kinetic with the maximum potential energy:

$$K_{\max} = \frac{1}{2}\,M\,\dot{\phi}^2\,L^2 = U_{\max} = M\,g\,L\,[1 - \cos(\phi_{\max})]\,.$$

CAPA seems to use the approximation $1 - \cos(\phi_{\max}) = \phi_{\max}^2/2$.

Damped SHM (7). With probability p, reduction factor r:

$$p = \exp[x\,\ln(r^2)]\,, \quad t = x\,T\,.$$