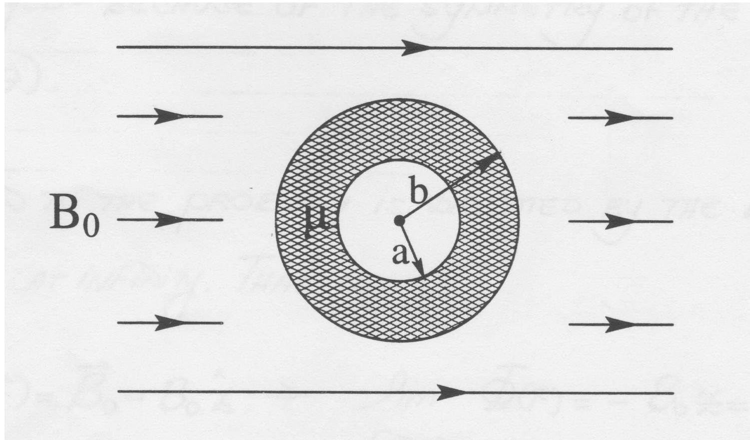

FIG. 1: Magnetism in matter: Concentric spheres in a magnetic field $\vec{B}_0 = B_0\,\hat{z}$.

Electrodynamics B (PHY 5347): Test on Homework April 20, 2017.

1. **Magnetism in matter (25%).**

   Consider two concentric spheres with radii $a < b$ placed in a (formerly) constant magnetic field $\vec{B}_0 = B_0\,\hat{z}$. The inner and outer regions are vacuum, while the region between the spheres is made of a material of permeability μ. See figure 1.

   (a) Write down the Maxwell equations and the boundary conditions of relevance for this problem.

   (b) Introduce a magnetic scalar potential and show the differential equations and boundary conditions that it satisfies.

2. **Covariant retarded Green function (25%).**

   Show that the explicitly covariant expression

$$G_r(x-y) = \frac{1}{2\pi}\,\theta(x^0-y^0)\,\delta[\,(x-y)^2\,]$$

   agrees with the retarded Green function

$$G_r(x-y) = \frac{\delta(\tau-\xi)}{4\pi\,\xi} = \frac{\delta(x^0-y^0-|\vec{x}-\vec{y}|)}{4\pi\,|\vec{x}-\vec{y}|}\ .$$

3. **Charge conjugation, parity and time reversal in electrodynamics (25%).**

   (a) State which of the following electrodynamic quantities are even (+) and which are odd (−) under operations of charge conjugation (C), parity (P), and time reversal (T): 1. The potential component A^0, 2. The current density $\vec{J}$, 3. The potential components $\vec{A}$, 4. The electric field $\vec{E}$, 5. The magnetic field $\vec{B}$. State the results as (C, P, T). Start from the assumption that the charge density ρ has $(C, P, T) = (-, +, +)$.

   (b) Express the following Lorentz scalars in terms of $\vec{E}$ and $\vec{B}$:

$$F^{\alpha\beta}\, F_{\alpha\beta} \quad \text{and} \quad {}^*F^{\alpha\beta}\, F_{\alpha\beta} \ .$$

   State their transformation behavior under (C, P, T).

4. **TM waves in a rectangular wave guide (25%).**

   Consider the propagation of TM waves in a rectangular wave guide with its cross section in the x-y-plane and of inner dimensions a, b. Assume perfectly conducting surfaces, e^{+ikz} for the axial dependence, and $e^{-i\omega t}$ for the time dependence.

   (a) Calculate E^z.

   (b) Write down the lowest cut-off frequency.