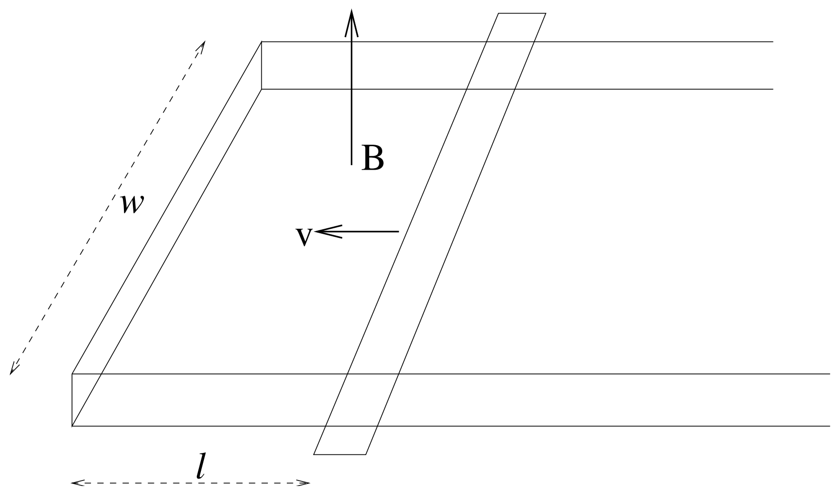
Electrodynamics B (PHY 5347) Winter/Spring 2017 Midterm, Feb. 23.

1. **Bar sliding in magnetic field (33%).**

   A bar slides along two parallel rails separated by a distance w, with no friction and with velocity v. The parallel bars, connector, and sliding bar are all made of a conductor with resistance r per unit length. The bars are in a uniform magnetic field B which is perpendicular to the plane of the bars and directed upward. Assume the bar starts at a distance l from the end.



   (a) Use Gaussian unit and calculate the emf in the loop.

   (b) Find the current in the bar as a function of time.

2. **Four-potential of a moving point particle (33%).**

   A pointlike particle with the charge of an electron q_e travels with velocity

$$\beta = \frac{v}{c} = \frac{1}{2}$$

   on the straight line $y^1 = \beta y^0 = vt$. Find the electromagnetic potentials which are after one picosecond, i.e., $t = 1\,[ps] = 10^{-9}\,[s]$, observed at the origin $\vec{x} = 0$.

   Take $3 \times 10^9\,[dm/s]$ (decimeter $[dm]$) for the speed of light, q_e as charge unit and give the results in $[q_e/dm]$. First, draw a Minkowski space picture.

   You may use

$$\Phi\left(x^0, \vec{x}\right) = \left[\frac{q}{\left(1 - \vec{\beta}\cdot\hat{n}\right)\, R}\right]_{\rm ret}, \quad \vec{A}\left(x^0, \vec{x}\right) = \left[\frac{q\,\vec{\beta}}{\left(1 - \vec{\beta}\cdot\hat{n}\right)\, R}\right]_{\rm ret}.$$

3. **Euler-Lagrange equation (33%).**

Consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}\, h_{\alpha\beta}\, h^{\alpha\beta} + h^{\alpha\beta}\,(\partial_\beta A_\alpha - \partial_\alpha A_\beta) - \frac{8\pi}{c}\, A_\alpha\, J^\alpha$$

where $h_{\alpha\beta}$ is a rank two tensor field, $A_\alpha(x)$ is a vector field and J_α an external current. All these fields are regarded to be independent, including $h_{\alpha\beta}$. Find the equations of motion from the Euler-Lagrange equations of relativistic fields.