Applied Monte Carlo Simulations to Particle Physics

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Introduction

Prior to the 1980s, high-energy physics had little to do with computational science, but now, through Monte Carlo simulations, scientists can model natural systems with random number generation.

This project deals primarily with simulating spins in a one-dimensional lattice. A lattice can resemble a string or loop of beads, or a higher dimensional grid of particles. In this case, the lattice resembles a string of particles, each with an electromagnetic 'spin' which can take on certain values; each spin has two allowed values, or states, for each particle. The potential energy of the lattice can be determined based upon the interaction of spins and their nearest neighbors. Each lattice has a particular boundary condition (BC); the two that are analyzed in this project are the open (linear) and periodic (circular) boundary conditions.

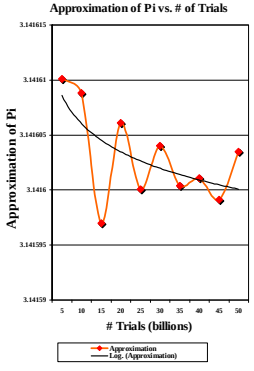
Hypothesis

Utilizing computer-based Monte Carlo methods, the algorithms will generate data sets in statistical agreement with the analytically calculated physics models.

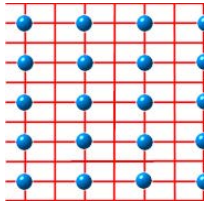
Outline of Procedures

1. Using the Marsaglia Pseudo-Random Number generator, the algorithm creates a configuration of spins (+ or -) on a one dimensional lattice grid.
2. Compare each spin with that of its neighbors. If the spins are the same, then increment the total energy of the system by one; otherwise decrement the energy by one. Repeat for every combination of two adjacent spins.
3. Divide the total energy by the number of lattice points to calculate the energy per spin.
4. Perform many 'sweeps' (touching each spin once) through the lattice. Repeat steps 2 and 3 after each sweep.
5. Compare the average energy per spin in the simulation to the analytically calculated result.
6. Change the parameter value of Beta (Boltzmann's constant divided by temperature, assume units such that Boltzmann's constant becomes one) and repeat steps 1-5.

An Elementary Monte Carlo Application



1. Generate two pseudo-random numbers (x and y coordinates) with the Marsaglia generator that fall within a unit square.
2. Test if this point is within a quarter unit circle centered on the lower left vertex of the square ($x^2 + y^2 \leq 1$).
3. Repeat steps 1 and 2 50,000,000,000 times, keeping track of the number of hits within the circle.
4. Pi ≈ 4 * the number of hits / 50,000,000,000



An example of a 2d lattice. A 1d open BC lattice would consist of one of these rows or columns. (Photo credit to http://www.clarku.edu/~djoyce/wallpaper/lattices.html)

Energy per Spin

(Periodic Boundary Conditions)

$$Z_n = 2^n(\cosh^n(\beta)+\sinh^n(\beta))$$

$$< E >= -\frac{d}{d\beta}\ln(Z)$$

$$< E >= -\frac{d}{d\beta}(n\ln(2)+\ln(\cosh^n(\beta)+\sinh^n(\beta)))$$

$$< E >= -\frac{\frac{d}{d\beta}(\cosh^n(\beta)+\sinh^n(\beta))}{\cosh^n(\beta)+\sinh^n(\beta)}$$

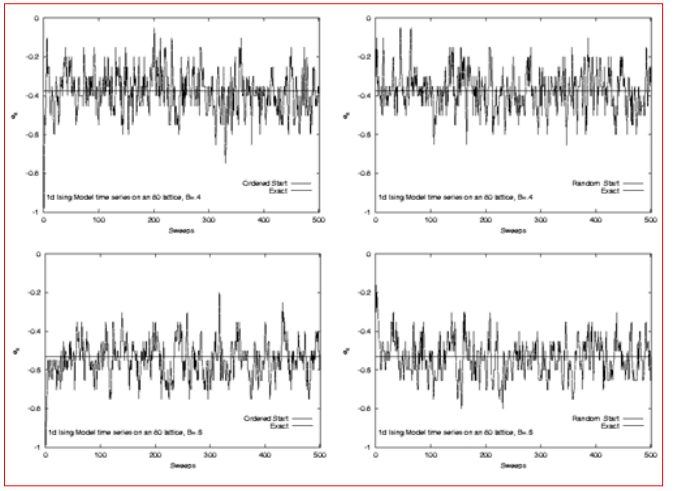
$$< E >= \frac{-n(\cosh^{n-1}(\beta)\sinh(\beta)+\sinh^{n-1}(\beta)\cosh(\beta))}{\cosh^n(\beta)+\sinh^n(\beta)}$$

$$< E >= \frac{-n(\frac{\sinh(\beta)}{\cosh(\beta)}+\frac{\sinh^{n-1}(\beta)}{\cosh^{n-1}(\beta)})}{1+\frac{\sinh^n(\beta)}{\cosh^n(\beta)}}$$

$$< E >= \frac{-n(\tanh(\beta)+\tanh^{n-1}(\beta))}{1+\tanh^n(\beta)}$$

$$< E_s >= \frac{-(\tanh(\beta)+\tanh^{n-1}(\beta))}{1+\tanh^n(\beta)}$$

Energy per Spin vs. Number of Sweeps

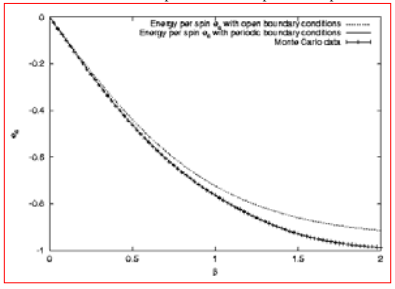


Analysis

Although the energy per spin in the graphs above appears to be scattered arbitrarily, it is centered around the analytically calculated value for that particular beta value. The graph below shows that with large numbers of sweeps, the Monte Carlo simulations correctly predict the energy per spin of systems with periodic BCs (with slight modifications to the code, the program would produce data for open BCs instead). As the beta value becomes especially large (and the corresponding temperature becomes very low), the Monte Carlo data will diverge slightly from the expected results. There is already evidence of deviation from the simulation in the open BC results. This can be thought of in terms of actual particles, which become increasingly elusive at micro-Kelvin temperatures. While this solution is fairly trivial in itself, this model does offer valuable insight into the more complex, higher-dimensional models, which have not yet been analytically solved.

Energy per Spin vs. Beta Value

Note: the Monte Carlo data produced corresponds to a periodic BC



Conclusion

The findings support the hypothesis that the Monte Carlo algorithms generate data sets in statistical agreement with the analytically calculated physics models.

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