Archimedes' Principle

The force exerted by a fluid on a body wholly or partially submerged in it is called the buoyant force.

- A body wholly or partially submerged in a fluid is buoyed up by a force equal to the weight of the displaced fluid.

Example: Figure 13-9 amd 13-10 of Tipler-Mosca.

Application: Density Measurements.

$$\rho = \frac{M}{V} = \frac{m_{\rm water} + \triangle W/g}{V_{\rm water}}$$

for a totally submerged object, where $\triangle W$ is the measured difference in the weight $W = M\,g$ of the object ($\triangle W$ may be negative).

Fluids in Motion

The general behavior of fluid in motion is very comples, because of the phenomen of turbulence. But there are some easy concepts governing the non-turbulent, steady-state flow of an incompressible fluid.

Continuity equation (Figure 13-13 of Tipler-Mosca):

Let v the velocity of the flow and A be the cross-sectional area, the

$$I_v = A\,v = \mathrm{constant}\ .$$

Bernoulli's Equation (Figures 13-14 and 13-15 of Tipler-Mosca):

$$P + \rho\,g\,h + \frac{1}{2}\rho\,v^2 = \mathrm{constant}\ .$$

PRS: In which part of a pipe will the pressure be lower?

1. The part with narrow cross-sectional area.

2. The part with large cross-sectional area.

Proof of Bernoulli's Equation:

We apply the work-energy theorem to a sample of fluid that initially is contained between points 1 and 2 in figure 13-14a. During time $\triangle t$ this sample moves to the region between points $1'$ and $2'$, see figure 13-14b. Let $\triangle V$ be the volume of the fluid passing point $1'$ during the time $\triangle t$, and $\triangle m = \rho \triangle V$ the corresponding mass. The same volume and mass passes point 2.

The net effect is that a mass $\triangle m$, initially moving with speed v_1 at height h_1 is transferred to move with speed v_2 at height h_2. The change of potential energy is thus

$$\triangle U = \triangle m\, g\,(h_2 - h_1) = \rho\, \triangle V\, g\,(h_2 - h_1)\ .$$

The change of kinetic energy is

$$\triangle K = \frac{1}{2} \triangle m \,(v_2^2 - v_1^2) = \frac{1}{2} \rho \triangle V \,(v_2^2 - v_1^2) \ .$$

The fluid behind the sample pushes with a force of magnitude $F_1 = P_1 A_1$ and does the work

$$W_1 = F_1 \triangle x_1 = P_1 A_1 \triangle x_1 = P_1 \triangle V \ .$$

The fluid in front of the sample pushes back with force $F_2 = P_2 A_2$ and does the negative work

$$W_2 = -F_2 \triangle x_2 = -P_2 A_2 \triangle x_2 = -P_2 \triangle V \ .$$

The total work done by these forces is

$$W_{\text{total}} = (P_1 - P_2) \triangle V = \triangle U + \triangle K$$

where the last equality is due to work-energy theorem (i.e. neglecting friction). Therefore, in this approximation

$$(P_1 - P_2) \triangle V = \rho \triangle V \, g \,(h_2 - h_1) + \frac{1}{2} \rho \triangle V \,(v_2^2 - v_1^2) \ .$$

Dividing $\triangle V$ out, moving all subscript 1 quantities to the left-hand side, and all subscript 2 quantities to the right-hand side give

$$P_1 + \rho\, h_1 + \frac{1}{2}\,\rho\, v_1^2 = P_2 + \rho\, h_2 + \frac{1}{2}\,\rho\, v_2^2$$

which can be restated as

$$P + \rho\, g\, h + \frac{1}{2}\,\rho\, v^2 = \mathrm{constant}\ .$$