ADVANCED DYNAMICS — PHY 4936

HOME AND CLASS WORK – SET 6

(November 1, 2011)

Read Landau-Lifshitz p.58 up to p.72 (§21 to §24).

(26) Continue with the double pendulum from assignment 25.

1. Use the eigenfrequencies $\omega_\pm$ given in the posted solution of 25 and normal coordinates (Landau-Lifshitz p.67/8) to write down the general solution for the two angles.
2. Express the integration constants of your solution through the angular positions and velocities at time $t = 0$, denoted by ϕ_0, $\dot\phi_0$, ψ_0, $\dot\psi_0$.
3. Use $\sqrt{l/g}$ as time unit and plot the solutions $\phi(t)$ and $\psi(t)$ up to $t = 50\sqrt{l/g}$ for initial conditions $\phi_0 = 0,\ \dot\phi_0 = \sqrt{g/l},\ \psi_0 = 0,\ \dot\psi_0 = -\sqrt{g/l}$.

Due November 7 before class (10 points).

(27) (A) Calculate the eigenfrequencies of a 2D harmonic oscillator

$$\left(\sum_{k=1}^{2} m_{ik}\,\ddot x_k + k_{ik}\,x_k\right) \;=\; 0\,, \quad (i = 1, 2)$$

with matrix elements

$$M \;=\; (m_{ik}) \;=\; \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \;\text{ and }\; K \;=\; (k_{ik}) \;=\; \begin{pmatrix} 5 & 1 \\ 1 & 2 \end{pmatrix}\,.$$

Due in class (4 points).

(B) Use normal co-ordinates Θ_1 and Θ_2 as defined in Landau-Lifshitz (p.67/68). Express x_1 and x_2 in terms of them. Due in class (4 points).

(C) Use the given numbers for m_{ik} and k_{ik} and write down the Lagrangian as function of $\dot x_1$, $\dot x_2$, x_1 and x_2. Then, substitute normal co-ordinates as found in (B) and write down the Lagrangian in terms of $\dot\Theta_1$, $\dot\Theta_2$, Θ_1 and Θ_2. Due November 7 before class (6 points).

(D) Assume at time $t = 0$ the initial conditions $\Theta_1(0) = 1$, $\dot\Theta_1(0) = 0$, $\Theta_2(0) = 0$ and $\dot\Theta_2(0) = 1$. Plot the resulting solution first in in the Θ_1-Θ_2 plane and then in the x_1-x_2 plane. Due November 9 before class (4 points).

(28) The Lagrangian of a 2D oscillator is

$$L \;=\; \frac{m}{2}\left(\dot x^2 + \dot y^2 - \omega_1^2\,x^2 - \omega_2^2\,y^2\right)\,.$$

Write down the general solution for the case that $x = y = 0$ at $t = 0$. Which condition applies to ω_1 and ω_2, so that the mass point returns to $x = y = 0$ at some future time t? How long will it take? Due November 14 before class (4 points).

Read Landau-Lifshitz p.96 up to p.101 (§31 and §32).