

SOLUTIONS MIDTERM PHY 4936 (Fall 2011)

PROBLEM 2

2.1 In spherical coordinates with $r = R = \text{const}$

$$L = \frac{m}{2} R^2 (\dot{\phi}^2 \sin^2 \theta + \dot{\theta}^2) - m g R \cos \theta .$$

2.2 The conservation laws are

$$E = \frac{m}{2} R^2 (\dot{\phi}^2 \sin^2 \theta + \dot{\theta}^2) + m g R \cos \theta$$

energy conservation and

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = m R^2 \dot{\phi} \sin^2 \theta = M_z .$$

angular momentum conservation.

3.3 Special solution $\theta(t) = \theta_0 = \text{const}$:

$$\dot{\phi} = \frac{M_z}{m R^2 \sin^2 \theta_0} = \omega_0 \Rightarrow \phi(t) = \omega_0 t + \phi_0 .$$