$t_2 > t_1$ <u>To show:</u>

$$\mathrm{Min} \int_{t_1}^{t_2} v(t)^2\, dt = (t_2 - t_1)\, \bar{v}^2$$

$$\bar{v} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} v(t)\, dt$$

<u>Proof:</u> "the mean square of something that deviates around an average, as you Know, is alway greater than the square of the mean" (Feynman 19-2)

$$0 \leq \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} \left[v(t) - \bar{v}\right]^2 dt =$$

$$(t_2 - t_1)^{-1} \int_{t_1}^{t_2} \left[v(t)^2 - 2v(t)\bar{v} + \bar{v}^2\right] dt =$$

$$(t_2 - t_1)^{-1} \int_{t_1}^{t_2} v(t)^2\, dt - 2\bar{v}\bar{v} + \bar{v}^2$$

$$\frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} \left[v(t)^2 - \bar{v}^2\right] dt$$

Minimum $= 0 \Rightarrow v(t) = \bar{v}$.