

$$L = \frac{m_1}{2}(\dot{x}_1)^2 + \frac{m_2}{2}(\dot{x}_2)^2 - k(x_1 - x_2)^2$$

$$\begin{aligned}\delta_x L &= \frac{\partial L}{\partial x_1} \delta x_1 + \frac{\partial L}{\partial x_2} \delta x_2 \\ &= -2k(x_1 - x_2) \delta x_1 + 2k(x_1 - x_2) \delta x_2 \\ &= -2k(x_1 - x_2) \delta x + 2k(x_1 - x_2) \delta x \\ &= 0\end{aligned}$$

where $\delta x_1 = \delta x_2 = \delta x$ because both particles are being translated by the same amount.

This shows that the sum of the forces in a closed system is 0. The force particle 1 exerts on particle 2 is equal and opposite to the force particle 2 exerts on particle 1.

$$L = \frac{m_1}{2} \dot{x}^2 - kx^2$$

$$\begin{aligned}\delta_x L &= \frac{\partial L}{\partial x} \delta x \\ &= -2kx \delta x\end{aligned}$$

This is not a closed system so the sum of the forces is not 0.