Solution for assignment 15:

The total Energy the is

$$E = \frac{1}{2}\,\dot{x}^2 + \frac{1}{2}\frac{k}{x^2}\ . \tag{1}$$

The initial values imply

$$E = \frac{1}{2}\frac{k}{x_0^2}\ . \tag{2}$$

Therefore,

$$\dot{x}^2 = \frac{k}{m}\left(\frac{1}{x_0^2} - \frac{1}{x^2}\right) \tag{3}$$

$$\dot{x} = \pm\sqrt{\frac{k}{m\,x_0^2}}\,\frac{\sqrt{x^2 - x_0^2}}{x}\ . \tag{4}$$

Separation of variables gives

$$\pm\sqrt{\frac{k}{m\,x_0^2}}\int_0^t dt' = \pm\sqrt{\frac{k}{m\,x_0^2}}\,t = \int_{x_0}^{x} dx'\,\frac{x'}{\sqrt{x'^2 - x_0^2}} = \sqrt{x^2 - x_0^2}\ . \tag{5}$$

Therefore, the solution is

$$x(t) = \sqrt{\frac{k}{m\,x_0^2}\,t^2 + x_0^2}\ . \tag{6}$$