ADVANCED DYNAMICS — PHY 4241/5227
HOME AND CLASS WORK – SET 6

Solution for assignment 26:

Double pendulum solution and plot (continuation of 25).

Let us take minors with respect to the first row of the determinant. For the ω_+ frequency the ratio of the two minors is

$$\frac{\triangle_{1+}}{\triangle_{2+}} = \frac{-1-\sqrt{2}}{2+\sqrt{2}} = \frac{(-1-\sqrt{2})}{(2+\sqrt{2})}\,\frac{(1-\sqrt{2})}{(1-\sqrt{2})} = \frac{1}{-\sqrt{2}}$$

and for ω_- it is

$$\frac{\triangle_{1-}}{\triangle_{2-}} = \frac{-1+\sqrt{2}}{2-\sqrt{2}} = \frac{(-1+\sqrt{2})}{(2-\sqrt{2})}\,\frac{(1+\sqrt{2})}{(1+\sqrt{2})} = \frac{1}{\sqrt{2}}\,.$$

Therefore, the solutions (real part) can be written

$$\begin{aligned}
\phi_+(t) &= A_+\cos(\omega_+ t) + B_+\sin(\omega_+ t)\,,\\
\phi_-(t) &= A_-\cos(\omega_+ t) + B_-\sin(\omega_+ t)\,,\\
\phi(t) &= \phi_+(t) + \phi_-(t)\,,\\
\psi(t) &= -\sqrt{2}\,\phi_+(t) + \sqrt{2}\,\phi_-(t)\,.
\end{aligned}$$

The four constants are determined by the four initial value, e.g., ϕ_0, $\dot\phi_0$, ψ_0, $\dot\psi_0$ at time $t=0$:

$$\begin{aligned}
\phi_0 &= A_+ + A_-\,, & \psi_0 &= \sqrt{2}\,(-A_+ + A_-)\,,\\
\dot\phi_0 &= \omega_+ B_+ + \omega_- B_-\,, & \dot\psi_0 &= \sqrt{2}\,(-\omega_+ B_+ + \omega_- B_-)\,,
\end{aligned}$$

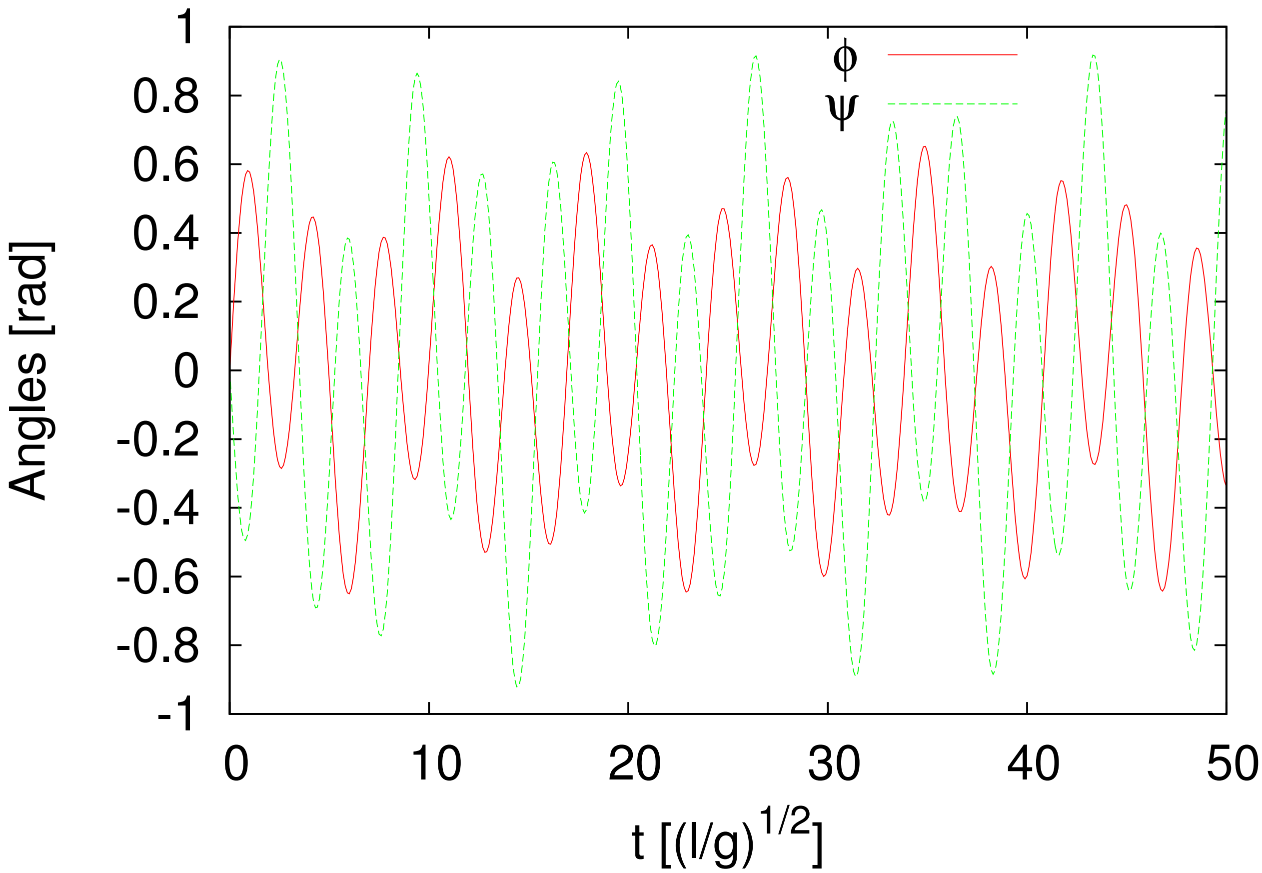
which gives

$$\begin{aligned}
A_+ &= \frac{\phi_0}{2} - \frac{\psi_0}{2\sqrt{2}}\,, & A_- &= \frac{\phi_0}{2} + \frac{\psi_0}{2\sqrt{2}}\,,\\
B_+ &= \frac{\dot\phi_0}{2\omega_+} - \frac{\dot\psi_0}{2\sqrt{2}\,\omega_+}\,, & B_- &= \frac{\dot\phi_0}{2\omega_-} + \frac{\dot\psi_0}{2\sqrt{2}\,\omega_-}\,.
\end{aligned}$$

For the initial conditions

$$\phi_0 = 0\,,\quad \dot\phi_0 = 1\,,\quad \psi_0 = 0\,,\quad \dot\psi_0 = -1$$

at time $t=0$ the figure gives a plot up to $t = 50\sqrt{l/g}$ (next page).



Angles [rad]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
10
20
30
40
50
t [(l/g)^{1/2}]
ϕ
ψ