Electrodynamics A (PHY 5346) Fall 2016 Final Decenber 14.

1. **Electron-positron annhilation (40%).**

   An electron and a positron, each (for $c = 1$) with mass equal to $0.511\,[MeV]$, annihilate at rest into two photons.

   (a) Working in the rest frame of the positronium, so that one of the photons moves in positive $x = x^1$ direction, find the momentum four-vector for each photon.

   (b) Suppose that the annihilation took place in a galaxy that is receding from us at $\beta = 3/5$. What is the energy of the photon that we observe?

2. **Coaxial cable (30%).**

   Consider a long, straight coaxial cable with a circular cross section. A constant current flows from a source to a target through the inner conductor ($0 \leq \rho \leq \rho_1$) and back through the outer conductor ($\rho_2 \leq \rho \leq \rho_3$). Assume vacuum in the in–between region ($\rho_1 < \rho < \rho_2$) and use Ampére's law to calculate the magnetic field everywhere.

3. **Electric potential from distinct BCs on half-spheres (30%).**

   (a) Consider a constant potential Φ_0 on the surface of the upper half of a hollow sphere of radius R and $\Phi = 0$ on the surface of the lower half of this sphere with no charges inside. Choose the center of the sphere at $x = y = z = 0$, its upper half above and its lower half below the $x - y$ plane. Use the Green function of a sphere to calculate the potential on the z axis inside this sphere.

   (b) Take $R = 1$ and $\Phi_0 = 1$. Evaluate $\Phi(z)$ for the values $z = R$, $z = R/2$, $z = 0$, $z = -R/2$ and $z = -R$. Sketch $\Phi(z)$ in the range $-1 \leq z \leq 1$.