Mathematical Physics — PHZ 3113

Classwork 8 (February 27, 2013)
Cylindrical Coordinates

1. Use cylindrical coordinates to calculate the area of a circle of radius R.

2. Calculate

$$\nabla \times \hat{z}\, \ln(\rho)$$

in cylindrical coordinates.

3. Show Oersted's law

$$\oint \vec{H} \cdot d\vec{r} \;=\; I$$

for the magnetic potential

$$\vec{A} \;=\; -\hat{z}\,\frac{\mu_0\, I}{2\pi}\,\ln(\rho)\,, \quad \vec{B} \;=\; \nabla \times \vec{A}\,,$$
$$\vec{H} \;=\; \mu_0^{-1}\vec{B}\,.$$

4. Find the acceleration $\vec{a}$ in cylindrical coordinates.

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Classwork 9 (March 1, 2013)

Cylindrical Coordinates

5. Completion of the square: We have

$$x^2 + b\,x + c$$

and want this in the form

$$x'^{\,2} + c' \ .$$

What are the values of x' and c'?

6. In cylindrical coordinates the equation of an ellipse is given by

$$\frac{p}{\rho} = 1 + \epsilon\,\cos(\phi)\,, \quad p > 0$$

with Cartesian coordinates $x = \rho\,\cos(\phi)$ and $y = \rho\,\sin(\phi)$. Assume $0 < e < 1$ for the eccentricity and transform the solution into the form

$$\frac{x'^2}{a^2} + \frac{y'2}{b^2} = 1 \ .$$

This means, derive the definitions of x', y', major half-axis a and minor half-axis b in terms of x, y, p and ϵ.

7. Use the definition

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1\,.$$

to calculate the area of an ellipse. Hint: Make a substitution, so that it becomes reduced to the area of a circle.