Mathematical Physics — PHZ 3113

Levi-Civita Tensor 1

(January 11, 2013)

Group #

Participating students (print):

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1. Use binary numbers 0, 1 and write down the numbers 0 to 3.

   0

   1

   2

   3

   Add one more column in which you substitute $0 \to 1$, $1 \to 2$ and one last column in which you count the decimal numbers from 1 to 4.

2. Use numbers with base 3 and symbols 0, 1, 2 to write down the numbers 0 to 26.

0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21

22
23
24
25
26

Add one more column in which you substitute $n \to n+1$ for $n = 0, 1, 2$ and one last column in which you count the decimal numbers from 1 to 27.

3. Write down the **permutations** of 1 2. How many permutations of 1 2 are there?
4. Write down the transposition of 1 2.
5. Use the permutations of 1 2 to write down the permutations of 1 2 3 by starting in each case with 3 on the right and transposing the number 3 with its left neighbor until this is no longer possible. How many permutations of 1 2 3 are there?
6. Along the same lines: Use the permutations of 1 2 3 to write down the permutations of 1 2 3 4. How many permutations

of 1 2 3 4 are there?

7. Proof that there are $n!$ permutions $\pi_1, \ldots, \pi_n$ of $1, \ldots, n$.

8. With $i_1 = 1, \ldots, n,\ i_2 = 1, \ldots, n,\ \ldots\ ,\ i_n = 1, \ldots, n$ the definition of the Levi-Civita tensor is

$$\epsilon_{i_1,\ldots,i_n} = \begin{cases} +1 \text{ for } i_1, \ldots, i_n \text{ even permutation,} \\ -1 \text{ for } i_1, \ldots, i_n \text{ odd permutation,} \\ 0 \text{ for } i_1, \ldots, i_n \text{ no permutation.} \end{cases}$$

   A permutation $i_1, \ldots, i_n$ is even, when it is generated by an even number of transpositions of $1, \ldots, n$ and odd, when it is generated by an odd number of transpositions of $1, \ldots, n$.

   How many non-zero elements are there?

   How many non-zero positive elements are there?

9. Write down all elements of the Levi-Civita tensor for $n = 2$.

10. Write down all elements of the Levi-Civita tensor for $n = 3$.

11. Write down all non-zero elements of the Levi-Civita tensor for $n = 4$.