Mathematical Physics — PHZ 3113

Levi-Civita Tensor 2 Applications

(January 14, 2013)

Group #

Participating students (print):

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1. In a **cyclic permutation** the first element becomes the last and the others stay in their order. Backward the last becomes the first and the others stay in their order.

   Write down the values of the cyclic permutations of ϵ_{123} and then of ϵ_{132}. Do you get all $3D$ values this way? Which are positive and which are negative?

2. Calculate in $3D$

$$\sum_{j=1}^{3}\sum_{k=1}^{3}\epsilon_{1jk}\,a_j\,b_k = \tag{1}$$

$$\sum_{j=1}^{3} \sum_{k=1}^{3} \epsilon_{2jk}\, a_j\, b_k = \tag{2}$$

$$\sum_{j=1}^{3} \sum_{k=1}^{3} \epsilon_{3jk}\, a_j\, b_k = \tag{3}$$

3. Calculate in $3D$

$$\sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} \epsilon_{ijk}\, \hat{x}_i\ a_j\, b_k = \tag{4}$$

and compare with $\vec{a} \times \vec{b}$.

Definition of the determinant of a nD matrix:

$$\begin{vmatrix} a_{11} & \dots & a_{1n} \\ . & & . \\ . & & . \\ . & & . \\ a_{n1} & \dots & a_{nn} \end{vmatrix} = \tag{5}$$

$$\sum_{i_1=1}^{n} \dots \sum_{i_n=1}^{n} \epsilon_{i_1 \dots i_n}\, a_{1i_1} \dots a_{ni_n} \,.$$

4. Calculate in $2D$

$$\sum_{i=1}^{2} \sum_{j=1}^{2} \epsilon_{ij}\, a_{1i} a_{2j} \;\; = \tag{6}$$

5. Calculate in $3D$

$$\sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} \epsilon_{ijk}\, a_{1i} a_{2j} a_{3k} \;\; = \tag{7}$$

using cyclic permutations.

6. Substitute in the previous expression

$$a_{11} = \hat{x}_1,\; a_{12} = \hat{x}_2,\; a_{13} = \hat{x}_3\,. \tag{8}$$

7. Substitute in the previous expression

$$a_{21} = a_1\,,\; a_{22} = a_2\,,\; a_{23} = a_3\,, \tag{9}$$

$$a_{31} = b_1\,,\; a_{32} = b_2\,,\; a_{33} = b_3\,. \tag{10}$$

8. In $3D$, write $\vec{a} \times \vec{b}$ as determinant (book p.23).

$$\vec{a} \times \vec{b} = \tag{11}$$

9. **Proof the $3D$ identity**

$$\sum_{i=1}^{3} \epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl} \tag{12}$$

by calculating the expression for the nine possibilites of values for jk, i.e., 11, 12, 13, 21, 22, 23, 31, 32, 33.