Mathematical Physics — PHZ 3113
Homework 7 (February 23, 2013)

1. Continuation of Midterm 1, Problem 5: Consider $\alpha_1 = \alpha_2 = \alpha$ and find the angle $\alpha_0 > 0$, so that for $0 < \alpha < \alpha_0$

$$\left|\vec{T}^{\,1}\right| = \left|\vec{T}^{\,2}\right| > \left|\vec{F}^{\,2}\right| \tag{1}$$

holds (6 points).

Solution: Let $T^1 = |\vec{T}^{\,1}|$, $T^2 = |\vec{T}^{\,2}|$ and $F = |\vec{F}|$. As $\alpha_1 = \alpha_2 = \alpha$ we have (second solution of the midterm)

$$T^1 \;=\; F\,\frac{\cos(\alpha)}{\sin(2\alpha)} \;=\; T^2$$

and we are looking for

$$1 \;=\; \frac{\cos(\alpha_0)}{\sin(2\alpha_0)} \;=\; \frac{\cos(\alpha_0)}{2\,\sin(\alpha_0)\,\cos(\alpha_0)}$$
$$=\; \frac{1}{2\,\sin(\alpha_0)} \;\Rightarrow\; \sin(\alpha_0) \;=\; \frac{1}{2}$$

$$\alpha_0 \;=\; \frac{\pi}{6}\,. \tag{2}$$

2. Use Stoke's Theorem to calculate the line integral of the previous homework (4 points).

Solution: The area of the triangle is

$$A = \frac{a\,h}{2} = \frac{a^2\sqrt{3}}{4}, \quad \vec{A} = A\,\hat{z}$$

and

$$\nabla \times \vec{F} = 2\,\hat{z}\,.$$

Using Stoke's theorem,

$$\oint_{\triangle} \vec{F} \cdot d\vec{s} = 2\,\hat{z} \cdot \vec{A} = \frac{a^2\sqrt{3}}{2}\,. \qquad (3)$$

Obviously, this is much shorter than the explicit calculation.