Mathematical Physics — PHZ 3113

Vectors 2 (Classwork January 9, 2013)

Group #

Participating students (print):

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1. Write down the **commutative** law of vector addition
$$\vec{a}+\vec{b} \;=\; \vec{b}+\vec{a}\,. \qquad (1)$$

2. Write down the **associative** law of vector addition
$$\left(\vec{a}+\vec{b}\right)+\vec{c} \;=\; \vec{a}+\left(\vec{b}+\vec{c}\right) \qquad (2)$$

3. How is the positively chosen angle θ between two nD vectors $\vec{a}$, $\vec{b}$ defined?
$$\cos(\theta) \;=\; \hat{a}\cdot\hat{b}\,, \quad 0\le\theta\le\pi\,. \qquad (3)$$

4. Write down the velocity for a nD position vector

$$\vec{r} \; = \begin{pmatrix} x_1(t) \\ . \\ . \\ . \\ x_n(t) \end{pmatrix} , \qquad \vec{v} \; = \begin{pmatrix} \frac{dx_1}{d\,t} \\ . \\ . \\ . \\ \frac{dx_n}{d\,t} \end{pmatrix} \tag{4}$$

5. Draw (millimeter paper provided)

$$\vec{r}(t) = \vec{r}_0 + \vec{v}\,t \tag{5}$$

with (in arbitrary units)

$$\vec{r}_0 = \begin{pmatrix} 2 \\ 4 \end{pmatrix} , \;\; \vec{v}_0 = \begin{pmatrix} 4 \\ 2 \end{pmatrix} , \;\; 0 \le t \le 2\,. \tag{6}$$

6. Calculate the work (in SI units $[J]$) for

$$\vec{F} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} [N] , \quad \triangle\vec{r} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} [m]\,. \tag{7}$$

$$W \; = \; (2+3)\,[N\,m] \; = \; 5\,[J]\,. \tag{8}$$

7. Describe the the surface swept out by $\vec{r}$ for

$$(\vec{r}-\vec{a})\cdot\vec{a} = 0\,, \tag{9}$$

$$(\vec{r}-\vec{a})\cdot\vec{r} = 0\,, \tag{10}$$

where $\vec{a}$ is a constant non-zero nD vector (compare exercise 1.2.2 of the book).

The trick is to write $\vec{r}$ as

$$\vec{r} = r_\perp\,\hat{a}_\perp + r_\parallel\,\hat{a} \quad \text{with} \quad \hat{a} = \frac{\vec{a}}{a} \tag{11}$$

where $a = |\vec{a}| > 0$ and $\hat{a}_\perp$ is any unit vector perpendicular to $\vec{a}$, i.e., $\hat{a}_\perp\cdot\hat{a} = 0$.

Calculation for (9):

$$\begin{aligned} 0 &= (\vec{r}-\vec{a})\cdot\vec{a} = \left(r_\parallel - a\right)\,a \\ \Rightarrow r_\parallel &= a \end{aligned} \tag{12}$$

and the solution is

$$\vec{r} = r_\perp\,\hat{a}_\perp + a\,\hat{a} \tag{13}$$

with arbitrary values for $r_\perp$. The is the $(n-1)\,D$ plane perpendicular to $\vec{a}$ and located at the tip of $\vec{a}$.

Calculation for (10):

$$0 \; = \; (\vec{r} - \vec{a}) \cdot \vec{r} \; = \; r_\perp^2 + \left(r_\| - a\right) r_\|$$

$$\Rightarrow r_\perp^2 \; = \; \left(a - r_\|\right) r_\|$$

with the solution

$$r_\perp \; = \; \pm\sqrt{\left(a - r_\|\right) r_\|} \qquad (14)$$

for $0 \le r_\| \le a$. The part $+\sqrt{\left(a - r_\|\right) r_\|}$ is drawn below (the $-\sqrt{}$ root is included by $\hat{a}_\perp \to -\hat{a}_\perp$).