JET OBSERVABLES IN THEORY AND REALITY

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Abstract

I discuss the one jet inclusive jet cross section, $d\sigma/dE_T$ emphasizing the concept of infrared safety and the cone definition of jets. Then I estimate the size of power corrections to the jet cross section, which become important at smaller values of E_T.

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Jet definitions and infrared safety

Jets are an obvious feature of the final state for $p+\bar{p}\to hadrons$. Although the jets are obvious, for a quantitative comparison of theory and experiment, one must carefully specify what one means by a jet. The jet definition associates with each jet a transverse energy E_T, a rapidity η, and an azimuthal angle ϕ. Then one can form the inclusive jet cross section

$$\frac{d\sigma}{dE_T} = \frac{1}{\eta_2-\eta_1}\int_{\eta_1}^{\eta_2} d\eta \int_0^{2\pi} d\phi \; \frac{d\sigma}{dE_T d\eta d\phi}. \tag{1}$$

With a suitable definition the cross section has the form

$$\frac{d\sigma}{dE_T\,d\eta\,d\phi} \approx \sum_{a,b}\int_{x_A}^1 d\xi_A \int_{x_B}^1 d\xi_B \;\; f_{a/A}(\xi_A,\mu)\;\; f_{b/B}(\xi_B,\mu)\;\frac{d\hat{\sigma}^{ab}(\mu)}{dE_T\,d\eta\,d\phi}, \tag{2}$$

where the $f_{a/A}(\xi,\mu)$ are the parton distributions and where $d\hat{\sigma}^{ab}/[dE_T\,d\eta\,d\phi]$ is the perturbatively calculated hard scattering function. (The lowest order is α_s^2; current calculations[1,2] reach α_s^3.)

The cross section should be insensitive to long times $\Delta t \gg 1/E_T$. However, we know that, long after the hard interaction, soft particles are emitted or absorbed. Thus, $d\sigma/dE_T$ should be insensitive to soft particles. We know also that long after the hard interaction a fast particle can split into two collinear particles or two collinear fast particles can join. Thus $d\sigma/dE_T$ should be insensitive to collinear splitting and joining. A measureable quantity such as a jet cross section that has these properties is said to be infrared safe.

I consider here the "Snowmass Accord" jet definition[3]. Define a jet cone of radius R in η-ϕ space. Typically, $R=0.7$. The jet variables $E_{T,J},\eta_J,\phi_J$ are defined in terms of the particles by

$$E_{T,J} = \sum_{i\in\mathrm{cone}} E_{T,i}\,, \tag{3}$$

$$\phi_J = \frac{1}{E_{T,J}}\sum_{i\in\mathrm{cone}} E_{T,i}\;\phi_i\,, \qquad \eta_J = \frac{1}{E_{T,J}}\sum_{i\in\mathrm{cone}} E_{T,i}\;\eta_i\,. \tag{4}$$

The cone axis must agree with the jet axis (η_J,ϕ_J). One iterates until agreement is reached.

There are some difficulties that require one to amend this jet definition.[4,5] First, sometimes the cones overlap. If there is more than some specified amount of transverse energy in the overlap region, then one merges the jets. Otherwise, one splits the transverse energy in the overlap region between the two jets. Second, there will be transverse energy in the jet cone that is not related to the high P_T parton that made the jet but instead is associated with the underlying event. Accordingly, one subtracts $\rho\,\pi R^2$ from E_T, where ρ is the average E_T per unit $d\eta\,d\phi$ in minimum bias events. At the Fermilab Tevatron, $\rho\,\pi(0.7)^2\approx 1.1$ GeV. Note that one does *not* do this in the corresponding order α_s^3 theoretical calculation since the underlying event is not part of the calculation.

In the theoretical calculation, one often modifies[6] the Snowmass defintion to restrict merging partons into jets according to a variable $R_{\rm sep}$. If two partons have

$$(\eta_1-\eta_2)^2+(\phi_1-\phi_2)^2 > R_{\rm sep}^2\,, \tag{5}$$

one does *not* merge them, even though

$$(\eta_i - \eta_{\rm jet})^2 + (\phi_i - \phi_{\rm jet})^2 < R^2, \qquad i = 1, 2. \tag{6}$$

Why? Studies[6] of the E_T distribution within jets suggest that the experimental jet algorithms fail to merge subjets that are rather far separated. The value $R_{\rm sep} = 1.3R$ is suggested. This lowers the theoretical cross section about 4%.

The theory for jet production comes with theoretical uncertainties. There are uncalcuated α_s^4 and higher perturbative contributions. A typical estimate[1] of this uncertainly is $\pm15\%$. In addition, there is uncertainty in the parton distributions. One may guess $\pm20\%$ for this uncertainty for $E_T < 300$ GeV. The uncertainty is presumably larger for $E_T > 300$ GeV because the gluon distribution is largely unknown[7] at large x. Finally, there are corrections suppressed by a power of "1 GeV"$/E_T$. In the remainder of this talk, I attempt to estimate these corrections using simple models.

Power suppressed corrections

In this section, I address power suppressed corrections to the theory. I will present graphs of *(Data - Theory) /Theory*. Both data and theory refer to the jet cross section $d\sigma/dE_T$ for $0.1 < |\eta| < 0.7$ and $R = 0.7$ as in Eq. (1). The data is from CDF[8] and has been corrected by CDF by subtracting from the jet E_T an estimate of the transverse energy contained in the jet cone that arises from the underlying event ($\Delta E_T \approx 1.1$ GeV). The theory here is straightforward order α_s^3 QCD with CTEQ4M partons. The theoretical calculation uses an $R_{\rm sep}$ correction to the Snowmass algorithm, with $R_{\rm sep} = 1.3R$.

Splash in. In α_s^3 theory, the transverse energy of a jet comes from one or two partons. It is the decay products of these partons (using the picture embedded in Monte Carlo models), that one wants to capture in the jet cone. However, other particles can splash into the cone. The underlying event creates soft particles that will get into the cone. An estimate of this effect has already been subtracted from E_T in the data, so we do not consider it further. In addition, the accelerated initial state partons radiate soft gluons, whose decay products can splash into the cone. Let us try to estimate this effect.

If transverse energy $\Delta E_T^{\rm in}$ is added to the jet E_T, the cross section should change by

$$\frac{d\sigma}{dE_T} \approx \left(\frac{d\sigma}{dE_T}\right)_{\rm NLO} \left\{1 + n\ \frac{\Delta E_T^{\rm in}}{E_T}\right\}, \tag{7}$$

where

$$n(\ln(E_T)) = -\frac{d}{d\ln(E_T)}\ \ln\left[\left(\frac{d\sigma}{dE_T}\right)_{\rm NLO}\right]. \tag{8}$$

The factor $1/E_T$ makes this a small correcton for large E_T. However, n is large.

Marchesini and Webber[9] have suggested a method for estimating from data the level of transverse energy in jet events without including the E_T from the third of three partons in a perturbative calculation. Lacking such a data-based analysis, I use Marchesini and Webber's Monte Carlo study for the amont of extra splash-in:

$$\Delta E_T^{\rm in} \approx 0.6\ \text{GeV}. \tag{9}$$

Splash-out. Some of the partonic transverse energy can leak out of the jet cone. The order α_s^3 perturbation theory gets this effect partly right: in a three parton final state the third parton can escape the jet cone. However, using the picture embedded in Monte Carlo models, the late stages of partonic branching and the final hadronization of the partons can also result in transverse energy escaping the jet cone. Here is a simple model for this effect.

Consider the hadrons that represent the decay products of a high E_T parton. Let η be the rapidity of the hadrons relative to jet axis. Let $\vec{k}_T$ be the transverse momentum of the particles relative to jet axis. Let the distribution of hadrons be

$$\frac{dN}{d\eta d\vec{k}_T} = \frac{A}{\pi \langle k_T^2 \rangle} \exp\left\{ -k_T^2 / \langle k_T^2 \rangle \right\}, \tag{10}$$

where A is the number of hadrons per unit rapidity and $\langle k_T^2 \rangle$ is average k_T^2 of the hadrons. Then the E_T lost is approximately

$$E_T^{\rm out} = \int_0^{\eta_1} d\eta \int d\vec{k}_T \, \frac{1}{2} |\vec{k}_T| e^{\eta} \, \frac{dN}{d\eta d\vec{k}_T}, \tag{11}$$

where $\eta_1 = -\ln\left(\tan(R/2)\right)$. Performing the integral gives

$$E_T^{\rm out} = \frac{\sqrt{\pi}}{4} A \sqrt{\langle k_T^2 \rangle} \left(e^{\eta_1} - 1\right). \tag{12}$$

Taking $\sqrt{\langle k_T^2 \rangle} = 0.3$ GeV and[10] $A = 5$, I find

$$E_T^{\rm out} \approx 1.1 \text{ GeV}. \tag{13}$$

This estimate can be used to correct the theoretical cross section, using the analog of Eq. (7) (with the opposite sign for $E_T^{\rm out}$ instead of $E_T^{\rm in}$).

In Fig. 1 below, I show the data compared to theory. In the same graph, I show a curve representing (*Modified Theory − Theory) / Theory*, where the modified theory includes a correction for splash-in/splash-out. Evidently, the correction does not improve the agreement between theory and experiment. The size of the correction provides some estimate of the theory error from this source.

Transverse momentum smearing. In an order α_s^3 calculation, incoming partons have zero transverse momentum. Thus the observed jet can recoil against only one or two jets. But in a more realistic model, the incoming partons can radiate multiple soft gluons, each carrying some transverse momentum. In addition, the partons in the proton wave function can have a "primordial k_T" by virtue of being bound in a hadron of finite size.

Let us try a simple model that accounts for these effects. There is a risk of double counting, since we may count the same gluon both as one of the multiple soft gluons and as one of the final state gluons in the hard scattering. We simply hope that the double counting effect will be small.

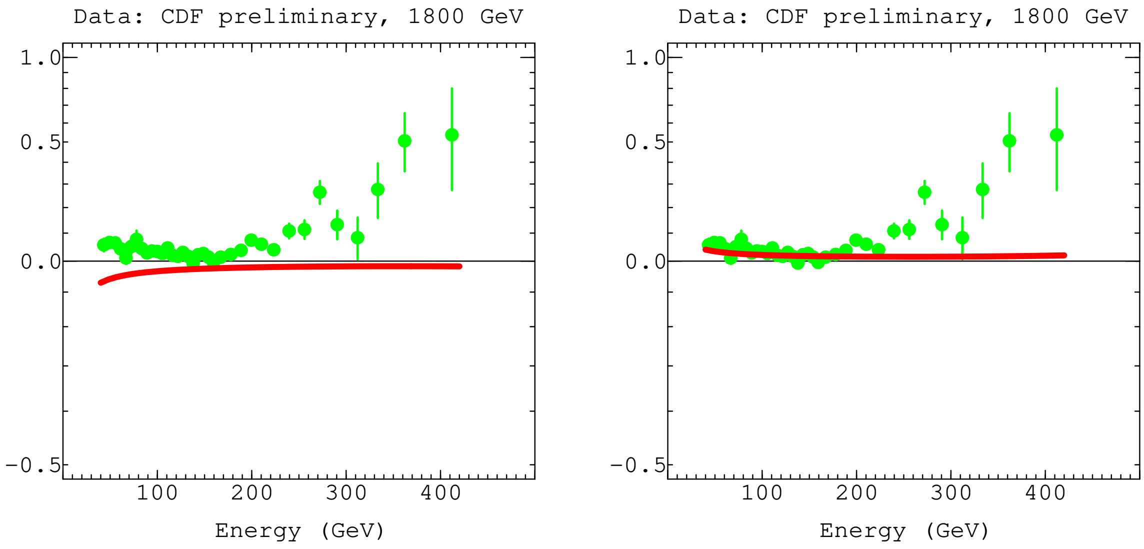

Figure 1: $(Data - Theory)/Theory$ for the jet cross section versus transverse energy E_T. In the left hand graph, the curve shows the correction to the theory for splash-in/splash-out. In the right hand graph, the curve shows the correction to the theory for k_T smearing.

Define a function $G(\vec{P}_T)$ that represents the measured cross section for one jet inclusive production:

$$G(\vec{P}_T) = \frac{d\sigma}{d\vec{P}_T} = \frac{1}{2\pi P_T}\,\frac{d\sigma}{dP_T}, \qquad P_T \equiv E_T. \tag{14}$$

Let $G\left(\vec{P}_T\right)_{\rm NLO}$ be the same cross section calculated at next to leading order. Then we can make a simple model for $G(\vec{P}_T)$:

$$G(\vec{P}_T) = \int d\vec{k}_T\ \rho(\vec{k}_T; P_T)\ G\left(\vec{P}_T - \frac{1}{2}\,\vec{k}_T\right)_{\rm NLO}, \tag{15}$$

where $\rho(\vec{k}_T; P_T)$ is a smearing function that represents the probability that the partons entering the hard scattering had transverse momentum $\vec{k}_T$. This function depends on P_T because large P_T in the hard scattering means more soft gluon radiation.

Supposing that $k_T \ll P_T$, we have

$$\begin{aligned} G(\vec{P}_T) \ \approx\ & \int d\vec{k}_T\ \rho(\vec{k}_T; P_T)\ \Big\{ G(\vec{P}_T)_{\rm NLO} \\ & -\frac{1}{2} k_T^i \nabla_i G(\vec{P}_T)_{\rm NLO} + \frac{1}{8} k_T^i k_T^j \nabla_i \nabla_j G(\vec{P}_T)_{\rm NLO} \Big\}. \end{aligned} \tag{16}$$

Define

$$n(\ln(P_T)) = -\frac{d}{d\ln(P_T)}\ \ln\left(2\pi P_T\ G(\vec{P}_T)_{\rm NLO}\right). \tag{17}$$

Then

$$\frac{d\sigma}{dE_T} \approx \left(\frac{d\sigma}{dE_T}\right)_{\rm NLO} \left\{1 + \frac{(n+1)^2 - n'}{16}\,\frac{\langle k_T^2\rangle}{E_T^2}\right\}. \tag{18}$$

To obtain a quantitative estimate, I use $n(\ln(E_T))$ from a NLO calculation and

$$\langle k_T^2\rangle \approx 2mE_T, \qquad m = 0.35 \text{ GeV}. \tag{19}$$

This formula is a rule of thumb. The number for m is based on identifying $2E_T$ with M_Z in $p + \bar{p} \to Z + X$, supposing that the k_T distribution of the Z is approximately gaussian for $k_T \ll M_Z$, and identifying $\langle k_T^2\rangle$ by noting that[11] $d\sigma/dk_T$ peaks at $k_T \approx 4$ GeV. Then

$$\frac{d\sigma}{dE_T} = \left(\frac{d\sigma}{dE_T}\right)_{\rm NLO} \left\{1 + \frac{(n+1)^2 - n'}{8}\,\frac{m}{E_T}\right\}. \tag{20}$$

(There is computer code by Baer and Reno[12] that does this sort of smearing for direct photon production, using a Monte Carlo style calculation.) In Fig. 1, I show the data compared to NLO theory with a correction for k_T smearing shown as a curve.

The net result. In Fig. 2, I show the data compared to the NLO theory with the net correction for splash-in/splash-out and k_T smearing shown as a curve.

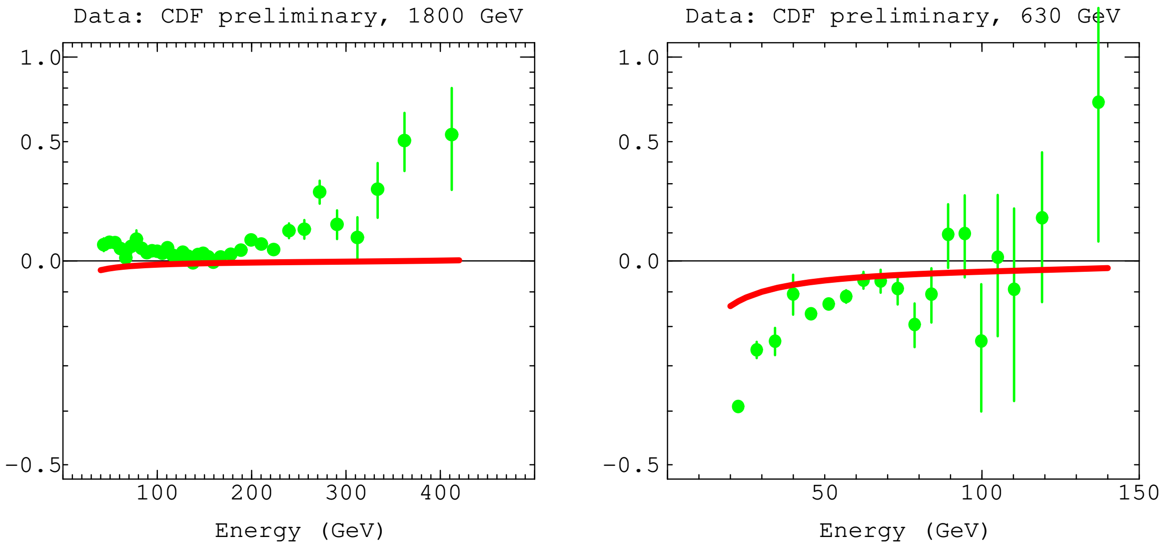

Figure 2: Graph of $(Data - Theory)/Theory$ versus E_T for with curves showing the net power supressed correction as estimated using Eqs. (7) and (20). The left hand graph is for $\sqrt{s} = 1800$ GeV while the right hand graph is for $\sqrt{s} = 630$ GeV

The net correction is quite small. If we take the size of the various corrections as an error estimate, we can estimate a theory error from power suppressed corrections of about $\pm 10\%$ at $E_T = 40$ GeV and $\pm 4\%$ at $E_T = 200$ GeV.

In Fig. 2, I also show the CDF data[13] at $\sqrt{s} = 630$ GeV compared to the NLO theory with the net correction for splash-in/splash-out and k_T smearing shown as a curve. I use the same parameters as for $\sqrt{s} = 1800$ GeV except that (estimating from the Monte Carlo study used earlier[9]), I reduce the splash-in parameter $E_T^{\rm in}$ from 0.6 GeV to 0.3 GeV.

If we take the size of the various corrections as an error estimate, then the estimated error is larger than at $\sqrt{s} = 1800$ GeV, perhaps $\pm 20\%$ at $E_T = 20$ GeV. It may be significant that, while the net effect at $\sqrt{s} = 1800$ GeV is really quite small, the net effect at $\sqrt{s} = 630$ GeV is not so small, and is in the right direction to improve the agreement between theory and experiment.

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