Loop precision
and
the advent of the Large Hadron Collider

Laura Reina

Loopfest VII, May 2008

- The Tevatron and even more the Large Hadron Collider (LHC) will test new ground and answer some of the fundamental open questions of Particle Physics:
  $\longrightarrow$ Electroweak (EW) symmetry breaking: Higgs mechanism?
  $\longrightarrow$ New Physics (NP) in the TeV range?
  $\longrightarrow$ ...

- With the LHC in particular we are finally moving from indirect constraints on NP to direct detection of NP! (see Landsberg's talk)

- The reach of the Tevatron and the incredible physics potential of the LHC relies on our ability of providing very accurate QCD predictions
  $\longrightarrow$ precise prediction of parameters (α_s, m_t, M_W, ...);
  $\longrightarrow$ precise prediction of signals/backgrounds. (several talks)

- With the LHC we will enter a new era of EW corrections: large 10-30% effects due to large logarithmic corrections. Interplay between QCD and EW corrections may be important. (see Kühn's talk)

Outline

- Overview.
- State of the art QCD corrections for hadron colliders:
  $\longrightarrow$ tracing progress at LO
  $\longrightarrow$ tracing progress at NLO
  $\longrightarrow$ tracing progress beyond NLO

  new techniques, breakthroughs, and perspectives.
- Highlights of EW results.
- What is left out ...
  $\longrightarrow$ flavor physics
  $\longrightarrow$ new physics corrections

Why pushing the Loop Order ...

- Stability and predictivity of theoretical results, since less sensitivity to unphysical renormalization/factorization scales. First reliable normalization of total cross-sections and distributions. Crucial for:
  $\longrightarrow$ precision measurements (M_W, m_t, M_H, $y_{b,t}$, ...);
  $\longrightarrow$ searches of new physics (precise modelling of signal and background);
  $\longrightarrow$ reducing systematic errors in selection/analysis of data.

- Physics richness: more channels and more partons in final state, i.e. more structure to better model (in perturbative region):
  $\longrightarrow$ differential cross-sections, exclusive observables;
  $\longrightarrow$ jet formation/merging and hadronization;
  $\longrightarrow$ initial state radiation.

- First step towards matching with algorithms that resum particular sets of large corrections in the perturbative expansion: resummed calculations, parton shower Monte Carlo programs.

When is NLO not enough?

- When NLO corrections are large, to tests the convergence of the perturbative expansion. This may happen when:

  $\longrightarrow$ processes involve multiple scales, leading to large logarithms of the ratio(s) of scales;

  $\longrightarrow$ new parton level subprocesses first appear at NLO;

  $\longrightarrow$ $\ldots$

- When truly high precision is needed (very often the case!).

- When a really reliable error estimate is needed.

$$\Downarrow$$

See examples to follow.

Main challenges . . .

- Multiplicity and Massiveness of final state: complex events leads to complex calculations. For a $2 \to N$ process one needs:
  - $\longrightarrow$ calculation of the $2 \to N+1$ (NLO) or $2 \to N+2$ real corrections;
  - $\longrightarrow$ calculation of the 1-loop (NLO) or 2-loop (NNLO) $2 \to N$ virtual corrections;
  - $\longrightarrow$ explicit cancellation of IR divergences (UV-cancellation is standard).

Virtual corrections remain the main hurdle!

- Flexibility of NLO/NNLO calculations via Automation:
  - $\longrightarrow$ algorithms suitable for automation are more efficient: could boost the reach for NLO/NNLO complex calculations;
  - $\longrightarrow$ forces the adoption of standards: improves communication and save time in comparisons;
  - $\longrightarrow$ faster response to experimental needs (think to the impact of projects like MCFM);
  - $\longrightarrow$ man power should be used to develop new methods and ideas!

State of the art of QCD calculations

for hadronic processes

Relative order	$2 \to 1$	$2 \to 2$	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4			NNLO	NLO	LO	
α_s^5				NNLO	NLO	LO

(from N. Glover)

Green light $\longrightarrow$ Done!

Red light $\longrightarrow$ Still work in progress!

NLO: very few $2 \to 3$ processes left, $2 \to 4$ barely touched ground (Figy,Hankele,Zeppenfeld, $H + 3j$, large N_c (08), Bozzi,Jäger,Oleari,Zeppenfeld: $VV + 2j$ via VBF (06-07))

NNLO: recent progress in $2 \to 2$ (Czakon, Mitov, Moch: $q\bar{q}, gg \to Q\bar{Q}$ at $O(m_Q^2/s)$ (07-08), Chachamis,Czakon: $q\bar{q} \to W^+W^-$ at $O(m_W^2/s)$ (08))

(plus: NNLO: NNLO splitting functions (Moch,Vermaseren,Vogt (04))).

Leading Order

highly automated but lacking precision

LO calculations can be used for qualitative studies, e.g. to qualitatively discriminate between different models or when high multiplicity is required.

And, they can be crucial in exploring new ground!

- automated generators of tree level matrix elements available:
  - ⟶ Feynman diagrams: CompHep/CalcHEP (Boos et al.), Madgraph/MadEvent (Maltoni et al.)+HELAS (Hagiwara et al.), SHERPA/AMEGIC++ (Krauss et al.)
  - ⟶ Off-shell recursion relations: ALPHA/ALPGEN (Caravaglios et al., Mangano et al.) , Helac (Kanaki et al.), VecBos (Giele)
  - ⟶ On-shell recursion relations: CSW (Cachazo, Svrček, Witten), BCFW (Britto, Cachazo, Feng, Witten), no public code available yet.
- automated integration over phase space;
- easier interface with parton shower.

Next-to-Leading Order

Parton level generators available for:

- all $2 \to 2$ processes
  well established results, both SM and MSSM. Many public codes.
- many $2 \to 3$ processes
  for both signal ($t\bar{t}H$, $b\bar{b}H$, H+2 jets, ...) and background ($t\bar{t}$+jet,V+2 jets, $Vb\bar{b}$, VV+jet, ...). Some public codes, mostly private codes.
- no $2 \to 4$ processes.

$\boxed{\text{HEPCODE database}}$ (http://www.cedar.ac.uk/hepcode/) database of available Monte Carlo codes, including LO, NLO and resummed predictions:

MCFM (Campbell and Ellis), AYLEN/EMILIA (Dixon, et al.), NLOJET++ (Nagy), JETRAD (Giele,et al.), FastNLO (Kluge, et al.), ResBos (Balazs, et al.), DIPHOX/EPHOX (Aurenche, et al.), VBFNLO (VBFNLO collaboration), ...

NLO: Recently completed calculations (since Les Houches 2005)

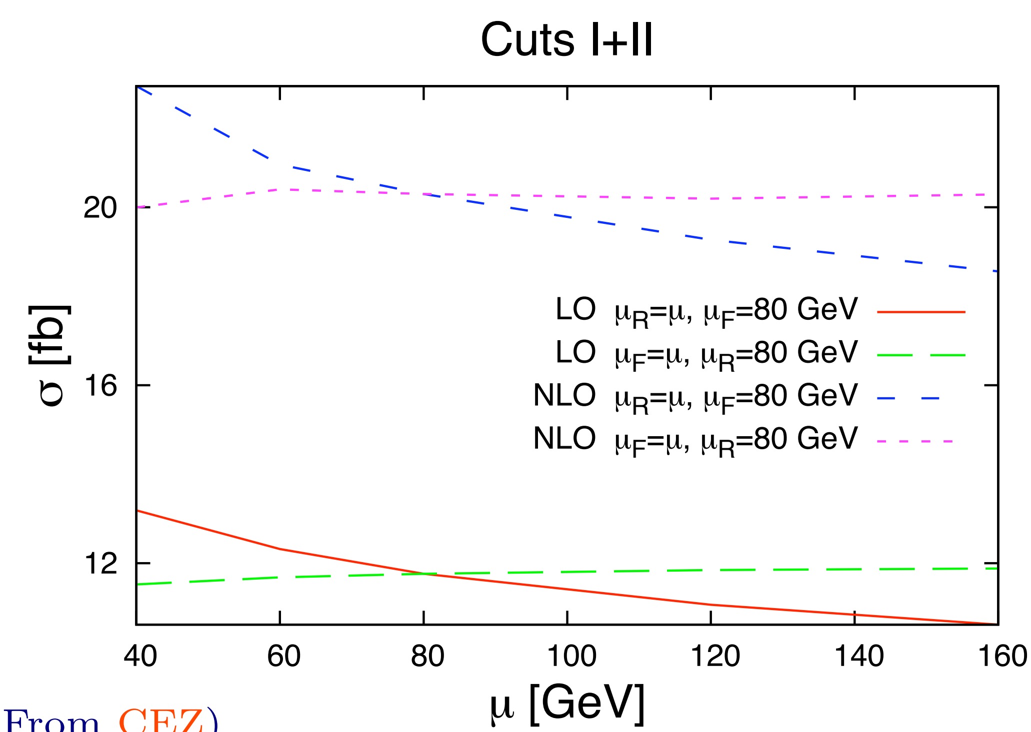
Process ($V \in \{Z, W, \gamma\}$)	Comments
$pp \to V$+2 jets(b)	Campbell,Ellis,Maltoni,Willenbrock (06)
$pp \to Vb\bar{b}$	Febres Cordero,Reina,Wackeroth (07-08)
$pp \to VV$+jet	Dittmaier,Kallweit,Uwer (WW+jet) (07)
	Campbell,Ellis,Zanderighi (WW+jet+decay) (07)
	Binoth,Karg,Kauer,Sanguinetti (in progress)
$pp \to VV$+2 jets	Bozzi,Jäger,Oleari,Zeppenfeld (via VBF) (06-07)
$pp \to VVV$	Lazopoulos,Melnikov,Petriello (ZZZ) (07)
	Binoth,Ossola,Papadopoulos,Pittau (WWZ,WZZ,WWW) (08)
	Hankele,Zeppenfeld ($WWZ \to 6$ leptons, full spin correlation) (07)
$pp \to H$+2 jets	Campbell,Ellis,Zanderighi (NLO QCD to gg channel)(06)
	Ciccolini,Denner,Dittmaier (NLO QCD+EW to VBF channel) (07)
$pp \to H$+3 jets	Figy,Hankele,Zeppenfeld (large N_c) (07)
$pp \to t\bar{t}$+jet	Dittmaier,Uwer,Weinzierl (07)
	Ellis,Giele,Kunszt (in progress)
$pp \to t\bar{t}Z$	Lazopoulos,Melnikov,Petriello (08)
$gg \to WW$	Binoth,Ciccolini,Kauer,Kramer (06)
$gg \to HH, HHH$	Binoth,Karg,Kauer,Rückl (06)

WW+jet: important background to Higgs searches in $gg \to H$ and VBF (with $H \to WW$)

Dittmaier,Kallweit,Uwer (07); Campbell,Ellis,Zanderighi (07)

Binoth,Karg,Kauer,Sanguinetti (in progress)

Les Houches 2007: comparison found good agreement among different calculations.

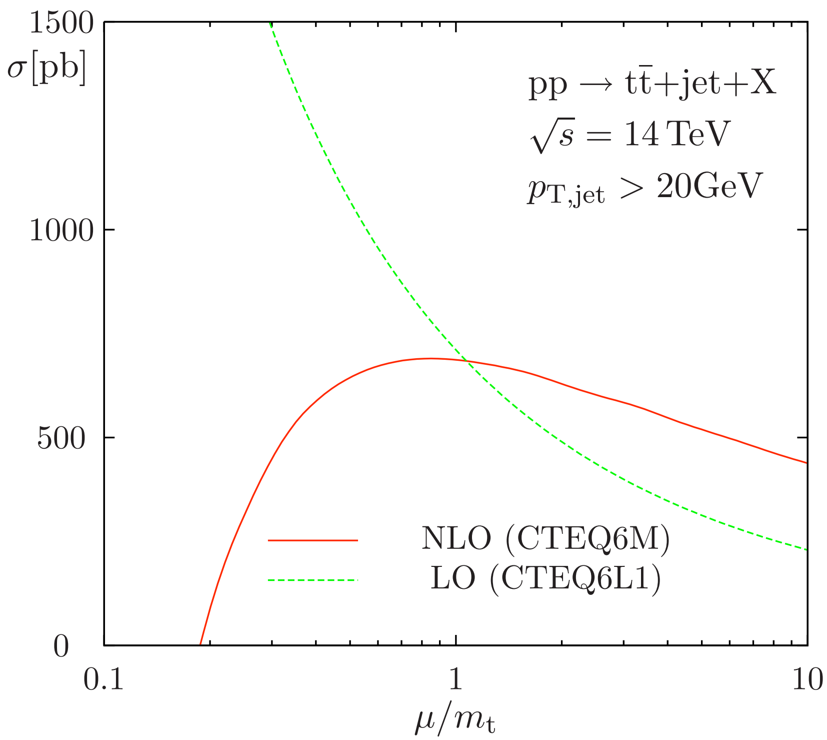


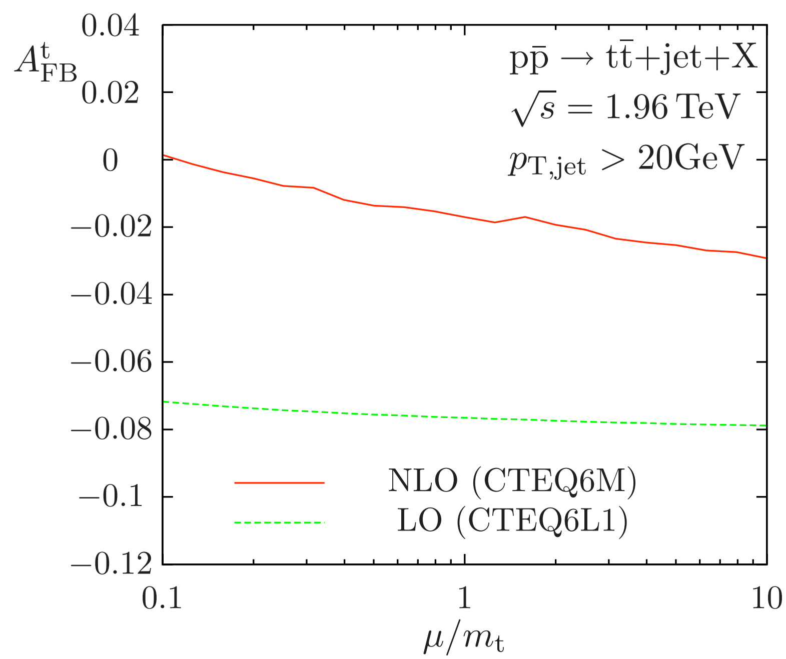
(From CEZ)

- $\longrightarrow$ include W (leptonic) decay
- $\longrightarrow$ cuts (II) discriminate signal from background
- $\longrightarrow$ 70% increase with NLO in selected region
- $\longrightarrow$ very mild residual scale dependence

$t\bar{t}$+jet: important background to Higgs searches in VBF and $t\bar{t}H$, more information on top quark properties, benchmark calculation

Dittmaier,Uwer,Weinzierl (07)





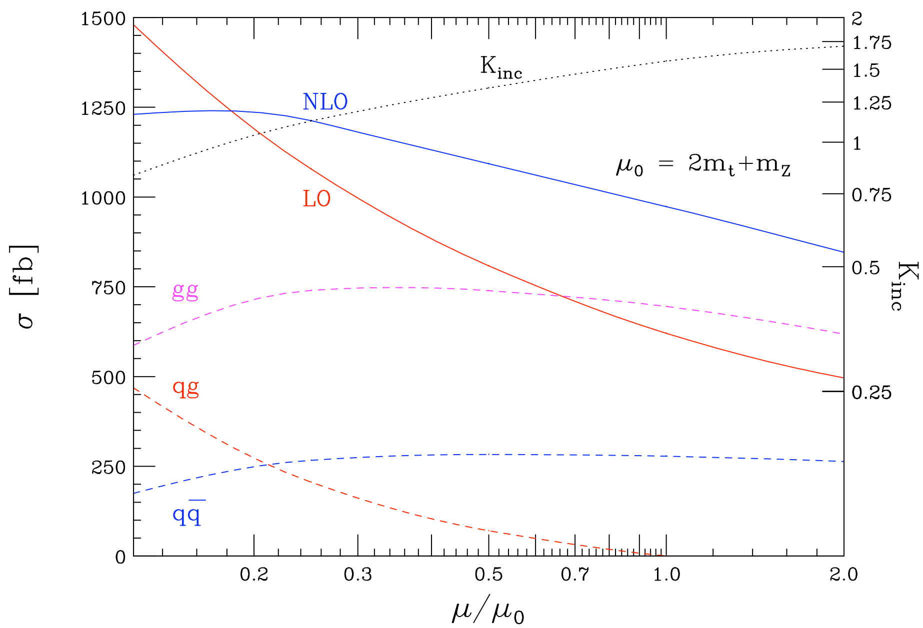
→ very reduced scale dependence;

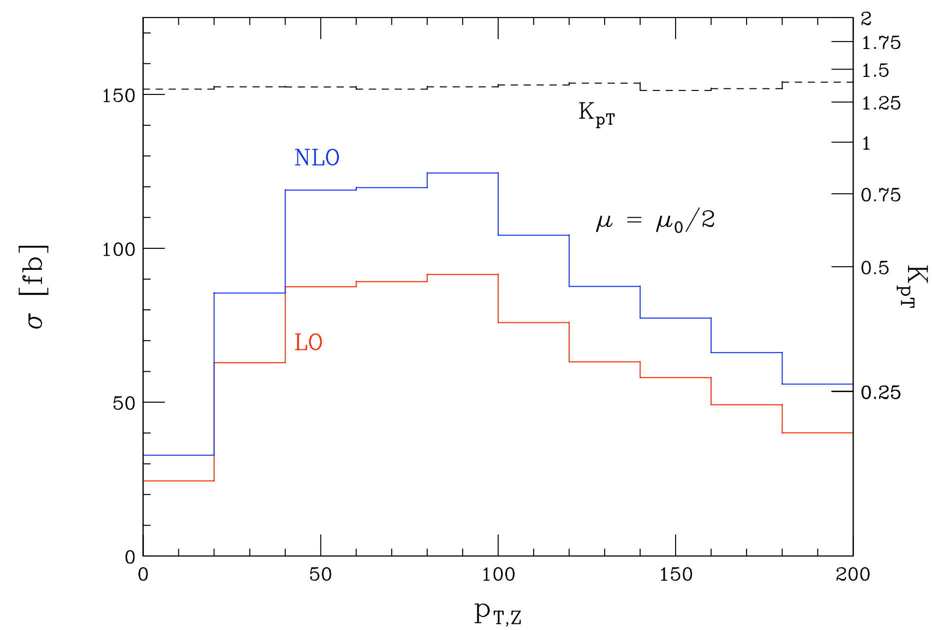
→ forward-backward asymmetry compatible with zero.

→ part of the NNLO corrections to $pp, p\bar{p} \to Q\bar{Q}$.

$Zt\bar{t}$: probing the top-quark electroweak properties and background to new physics (SUSY tri-lepton signatures)

Lazopoulos,McElmurry,Melnikov,Petriello (08)





→ very reduced scale dependence, about 11%;

→ large NLO corrections, minor impact on p_T^Z-distribution shape;

→ factor of 1.5-2 improvement with respect to LO analysis of couplings;

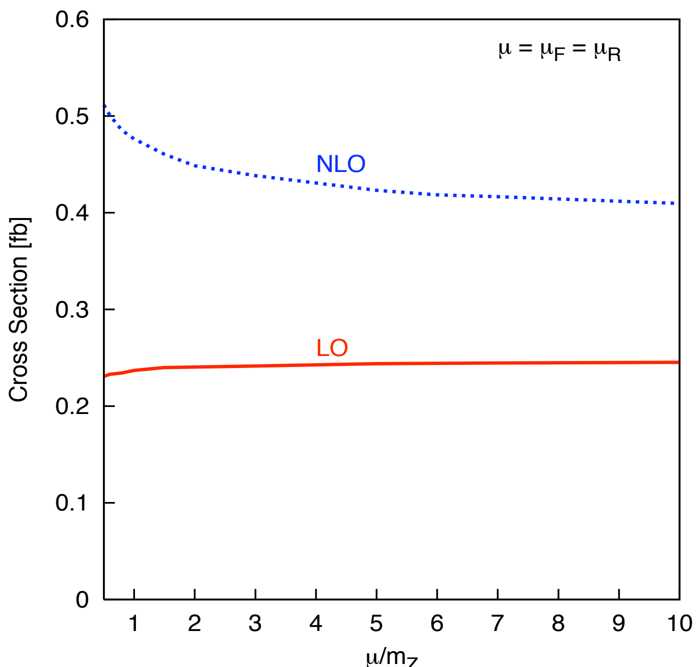
→ fully numerical calculation of one-loop matrix elements via sector decomposition and contour deformation.

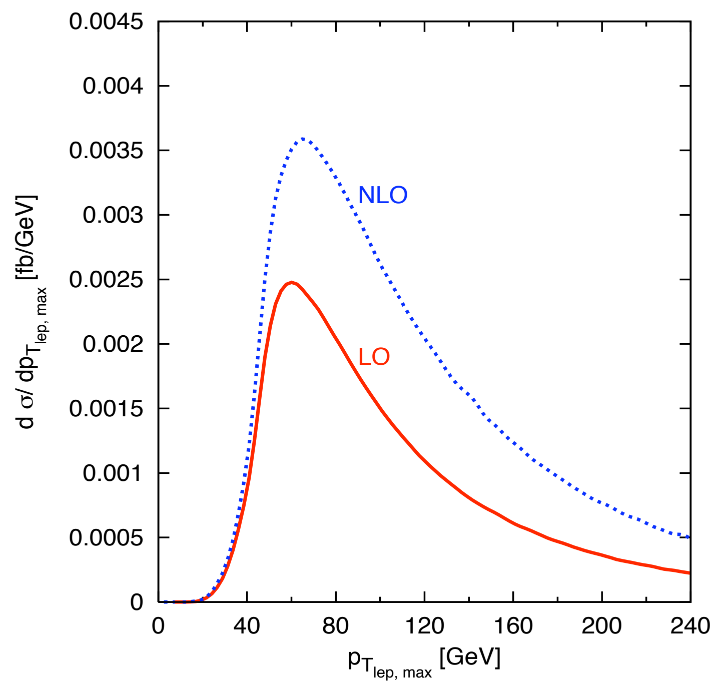
VVV: probing EW gauge couplings and background to new physics (SUSY leptons+$\not{p}_T$ signatures)

Hankele,Zeppenfeld (WWZ) (07)

Lazopoulos,Melnikov,Petriello (ZZZ) (07)

Binoth,Ossola,Papadopoulos,Pittau (WWZ,WZZ,WWW,ZZZ) (08)



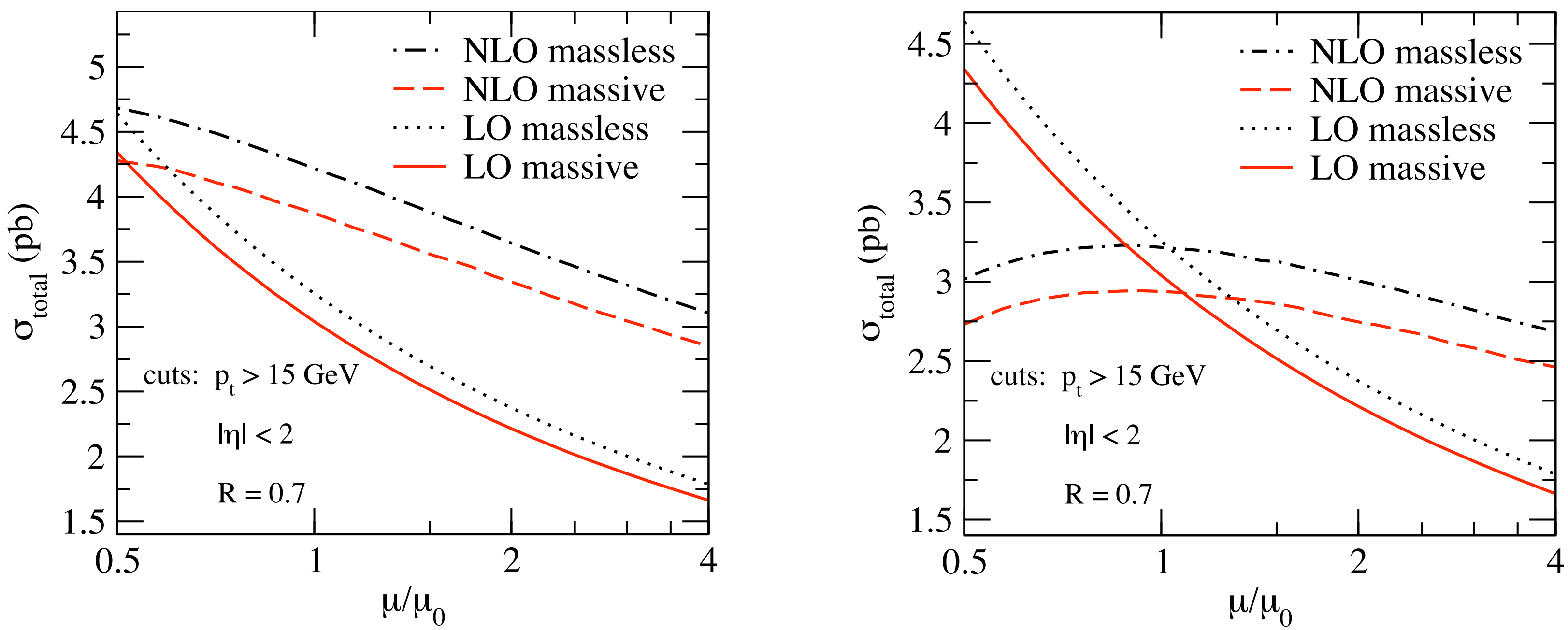


- $\longrightarrow$ fully spin correlated 6 lepton final state;
- $\longrightarrow$ LO uncertainty (about 1.7%) largely underestimated: no α_s dependence plus region of low μ_f dependence in PDFs;
- $\longrightarrow$ large NLO corrections on total cross section(K=1.7). At NLO theoretical uncertainty about 7.7%. Large impact on distribution shapes.
- $\longrightarrow$ available in VBFNLO and soon in KITCup collection.

$W/Zb\bar{b}$: main background to $W/Z + H$ production and single-top production

Febres Cordero, Reina, Wackeroth (07-08)

Zbb:



→ fully massive calculation ($m_b \neq 0$);

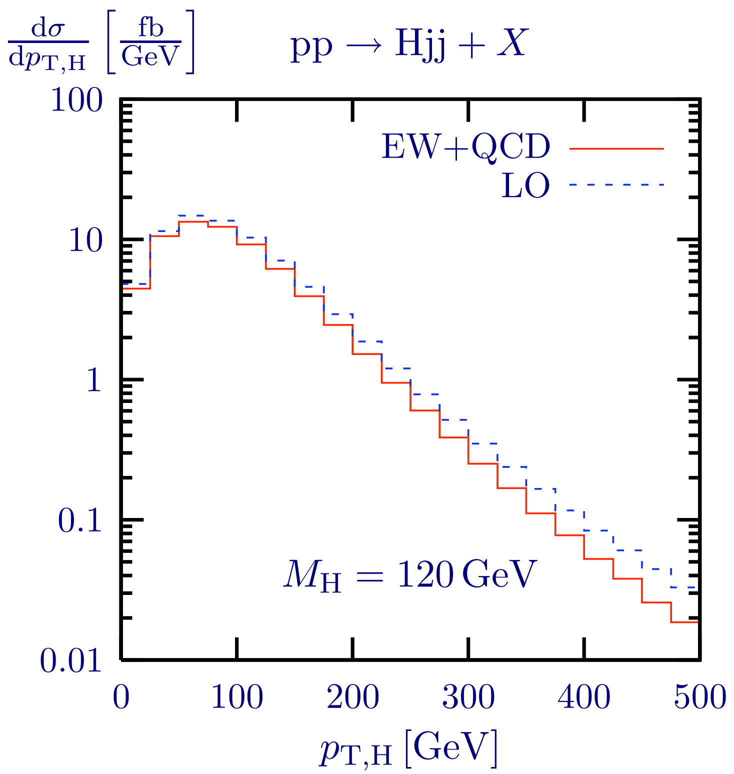
→ main effects in low $m_{b\bar{b}}$ region;

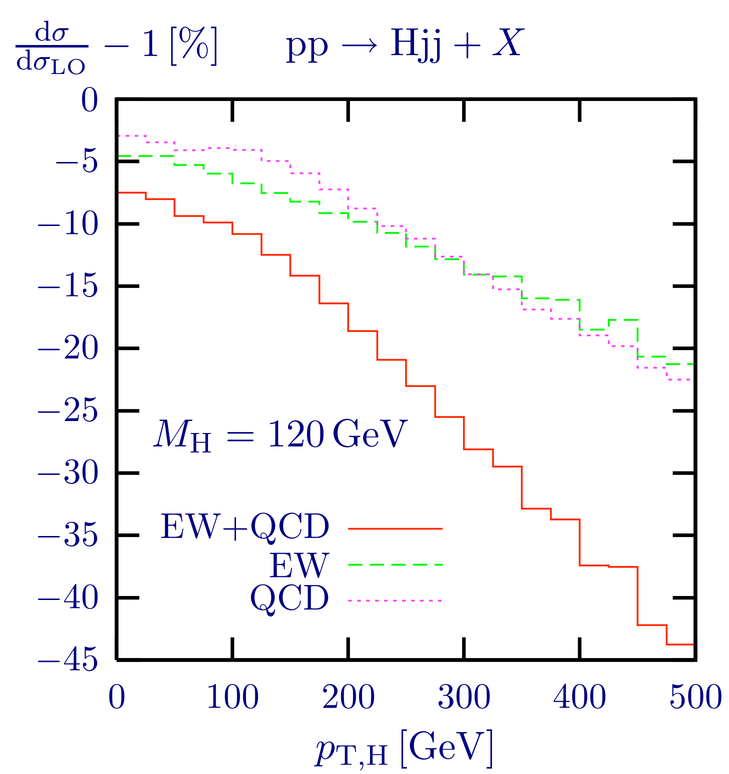
→ residual theoretical uncertainty: inclusive (20%), exclusive (10%);

→ partially tested using generalized unitarity methods.

EW and QCD corrections to Higgs boson production via VBF at the LHC

Ciccolini, Denner, Dittmaier (07)





- complete EW+QCD corrections to H+2j, including interferences;
- EW and NLO QCD corrections are of same size (5-10%) (!).

⟶ VBF-GF interference effects studied (see Jäger's talk).

Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations remaining from Les Houches 2005	
$pp \to t\bar{t}\,b\bar{b}$	relevant for $t\bar{t}H$
$pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$
$pp \to VV\,b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
$pp \to VV$+2jets	relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi),Jäger,Oleari,Zeppenfeld (06-07)
$pp \to V$+3jets	various new physics signatures
$pp \to b\bar{b}b\bar{b}$	Higgs and new physics signatures (see Reiter's talk)
Calculations beyond NLO added in 2007	
$gg \to W^*W^*$ $\mathcal{O}(\alpha^2\alpha_s^3)$	backgrounds to Higgs
NNLO $pp \to t\bar{t}$	normalization of a benchmark process
NNLO to VBF and Z/γ+jet	Higgs couplings and SM benchmark
Calculations including electroweak effects	
NNLO QCD+NLO EW for W/Z	precision calculation of a SM benchmark

NLO: computing the one-loop matrix elements

The one-loop amplitude of a generic $2 \to n$ process can be written as:

$$\mathcal{M}_n = \sum_i d_i I_4^i + \sum_i c_i I_3^i + \sum_i b_i I_2^i + \sum_i a_i I_1^i = C_n + R_n$$

where

$\longrightarrow$ I_4^i, I_3^i, I_2^i, I_1^i $\longrightarrow$ 4-,3-,2-, and 1-point 1-loop scalar integrals.
(known analytically for 1-loop QCD: QCDLoop, call EZ enterprise (07))

$\longrightarrow$ d_i, c_i, b_i, a_i $\longrightarrow$ process dependent D-dimensional coefficients.
Problem: compute these coefficients in an efficient and stable manner, i.e. reducing the overgrowing number of terms they consist of and avoiding numerical instabilities due to huge cancellations.

$\longrightarrow$ C_n: cut constructable part; R_n: rational terms.

Lots of new developments!

Very promising opening towards AUTOMATION!

- Amplitude via Feynman diagram representation:
  - $\longrightarrow$ improved tensor-integral reduction to avoid instabilities (Denner,Dittmaier; Binoth,Guillet,Heinrich, Pilon,Schubert; ...)
  - $\longrightarrow$ numerical loop integration, numerical evaluation of recursion relations (Nagy,Soper; Giele,Glover; Ellis,Giele,Zanderighi; ...)
  - $\longrightarrow$ sector decomposition plus contour deformation (Binoth,Heinrich; Anastasiou,Melnikov,Petriello; Soper; Lazopoulos,Melnikov,Petriello; Anastasiou,Beerli,Daleo)

- Amplitude via analytical structure: reconstruct the amplitude from its poles and branch cuts (using complex kinematics).
  - $\longrightarrow$ poles: factorize the amplitude into products of amplitudes with less external legs;
  - $\longrightarrow$ branch cuts: reduce the number of loop integrations (in conjunction with 4-d or D-d unitarity).

Rational terms (i.e. cut non-constructable terms) determined using various methods.

(Bern,Dixon,Dunbar,Kosower; Britto,Cachazo,Feng,Witten; Kilgore; Ossola,Papadopoulos,Pittau; Ellis,Giele,Kunszt,Melnikov; Anastasiou,Kunszt,Mastrolia; Catani,Gleisberg,Krauss,Rodrigo,Winter;...)

Aiming at Automation:

- Traditional packages for (partial) automation of 1-loop amplitudes include: FeynArts, QGRAF, FeynCalc, FF, FormCalc, Looptools, . . .

  (Hahn,Perez-Victoria,Nogueira,van Oldenborgh, Vermaseren, . . .)

  variously combined with in-house codes written in Form, Mathematica, Maple, ...

$$\Downarrow$$

- Move beyond analytic calculations of specific processes, aiming at complete, efficient and numerically stable computer codes:
  - $\longrightarrow$ OPP method implemented in `CutTools` (Ossola,Papadopoulos,Pittau)
  - $\longrightarrow$ generalized unitarity cuts plus on-shell recursion relations implemented in `BlackHat` (Berger,Bern,Dixon,Febres Cordero,Forde,Ita,Kosower,Maître)
  - $\longrightarrow$ numerical unitarity formalism, implemented in `Rocket` (Ellis,Giele,Kunszt,Melnikov; Giele,Zanderighi)

  Testing ground: $gg \to gggg$ (and more), $\gamma\gamma \to \gamma\gamma\gamma\gamma$ (see Bernicot's talk) .

- To be interfaced with real corrections: Catani-Seymour dipole subtraction method now implemented in automatic fashion.

  (Gleisberg,Krauss; Seymour,Tevlin)

NLO matching to Parton Shower Monte Carlos

... avoiding double counting of: collinear real emission; virtual corrections in Sudakov form factors.

$\longrightarrow$ MC@NLO (Frixione,Webber):

- W/Z boson production (Frixione,Webber (02));
- WW, ZZ, WZ boson pair production (Frixione,Webber (02));
- $gg \to H$ inclusive Higgs boson production (Frixione,Webber (02));
- $Q\bar{Q}$ heavy quark production (Frixione,Nason,Webber (03));
- single-top production (Frixione,Laenen,Motylinski,Webber (06)).

$\longrightarrow$ POWHEG (Nason)

- ZZ production (Nason,Ridolfi);
- $Q\bar{Q}$ heavy quark production (Frixione,Nason,Ridolfi);
- W/Z production (Alioli,Nason,Oleari,Re (08));
- Higgs boson production, single-top production, W/Z+jet (Alioli,Nason,Oleari,Re, (work in progress)).

$\longrightarrow$ Several proposal of new NLO matching to shower algorithms in progress. (Nagy,Soper; Schumann,Krauss; Giele,Kosower,Skands; ...)

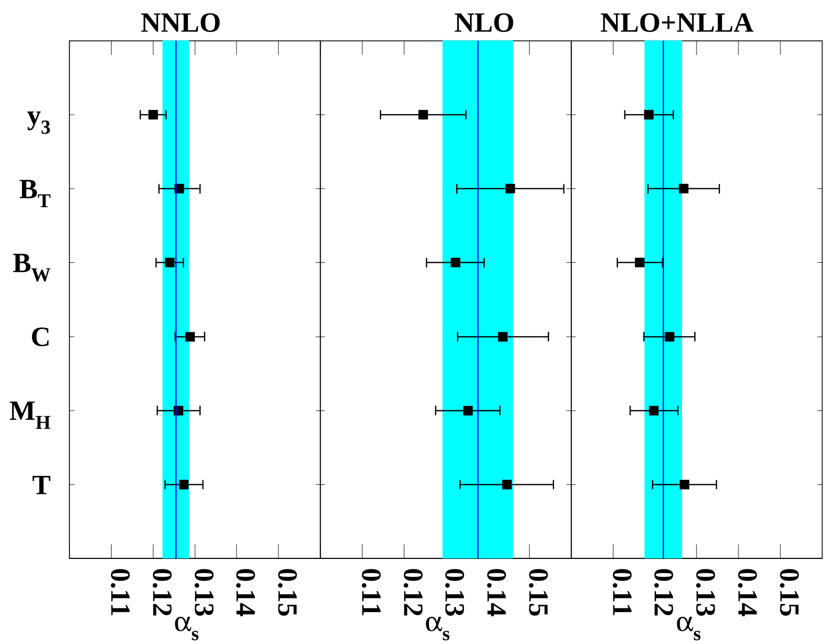
Next-to-Next-to-Leading Order

- Complexity of NNLO calculation requires new methods for both virtual and real corrections:
  - virtual: reduction to master integrals, Mellin-Barnes representation, nested sums, ..., (see talks by Czakon, Chachamis, Blümlein, ...)
  - real: subtraction method, sector decomposition (see Grazzini's talk)
- Pioneering calculations have started bringing extremely accurate results for some crucial physical observables, right at the advent of the LHC!
  - α_s from QCD observables in $e^+e^- \to 3$ jets (see Gehrmann's talk);
  - W/Z production: total and differential cross sections;
  - Higgs production: $gg \to H$, $b\bar{b} \to H$, WH/WZ associated production (see Grazzini's talk for $gg \to H, H \to WW, ZZ, \gamma\gamma$);
  - $Q\bar{Q}$ production ($Q = c, b, t$) (in $M_Q^2/s \to 0$ approximation) (see Czakon's and Mitov's talks);.

Each one a compelling physical case!

$e^+e^- \longrightarrow$ 3 jets, determining α_s via NNLO predictions for hadronic event shapes in e^+e^- annihilations.

Gehrmann-De Ridder,Gehrmann,Glover,Heinrich (07)



– pioneering calculation (e.g. real emission via antenna method)

– results more self-consistent (more precise and very little spread)

– larger than world average but ...

– wait for NNLO+NNLA!

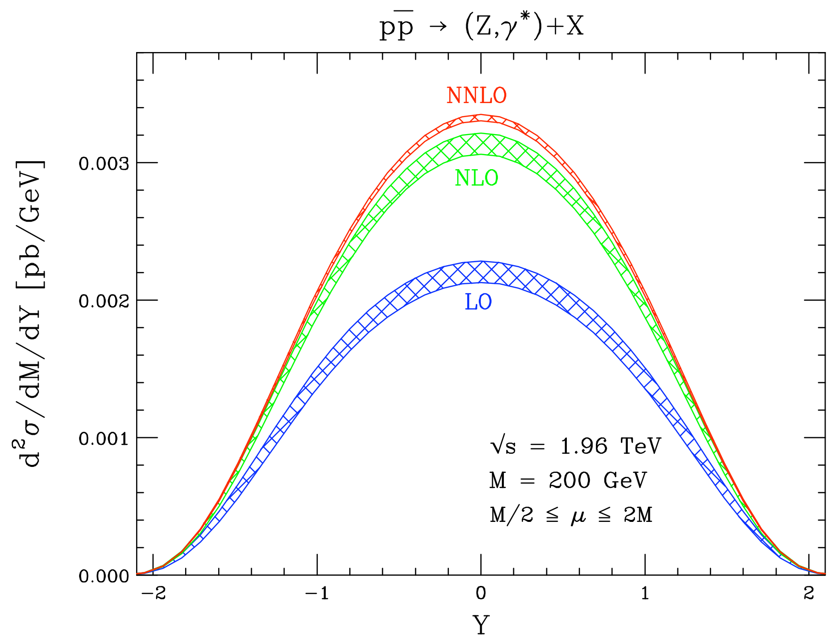
$$\alpha_s(M_Z^2) = 0.1240 \pm 0.0008\ (\text{stat}) \pm 0.0010\ (\text{exp}) \pm 0.0011\ (\text{had}) \pm 0.0029\ (\text{theo.})$$

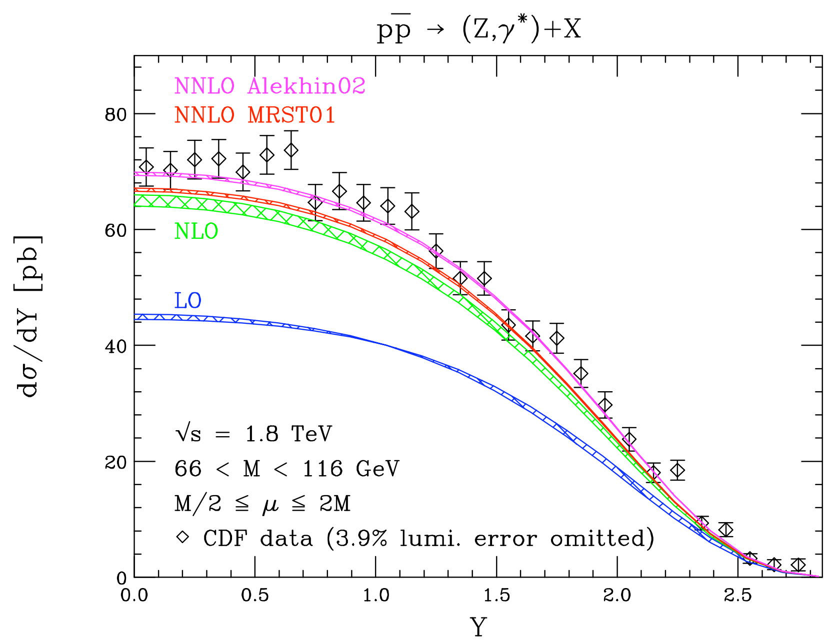
EW corrections: $O(\alpha_s \alpha_e^3)$ sizable, necessary for ILC precision, now calculated (Carloni Calame,Moretti,Piccinini,Ross)

W/Z production at the Tevatron and LHC, testing PDF's at NNLO.

Anastasiou,Dixon,Melnikov,Petriello (03)

Rapidity distributions of W and Z boson calculated at NNLO:

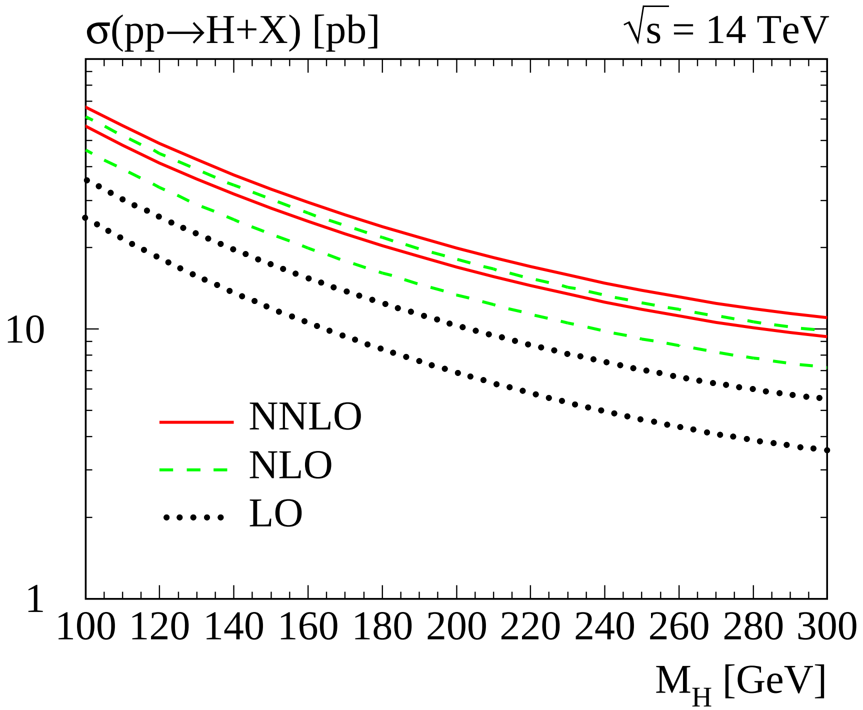


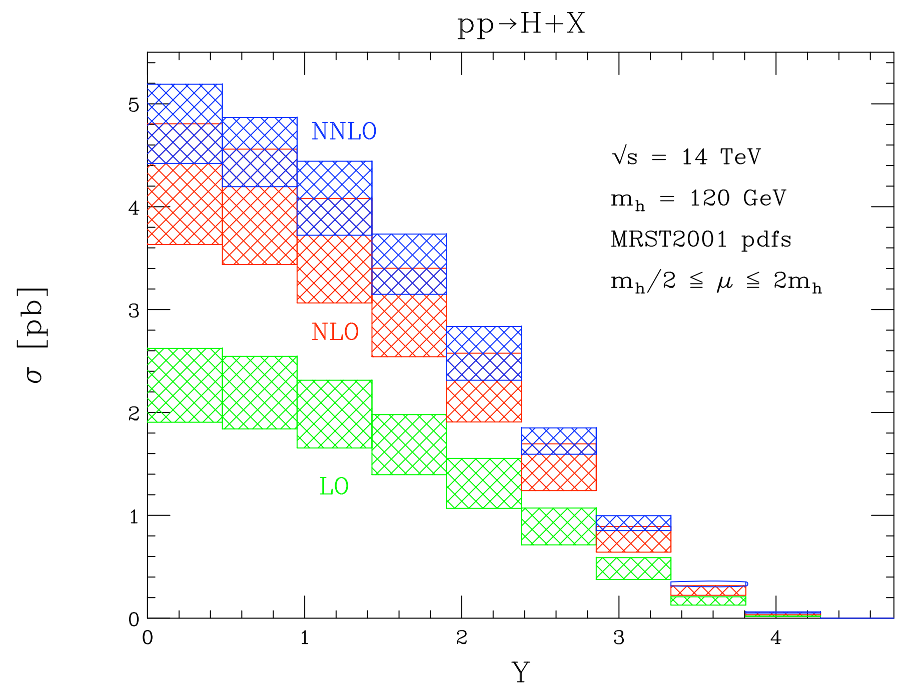


- W/Z production processes are standard candles at hadron colliders.
- Testing NNLO PDF's: parton-parton luminosity monitor, detector calibration (NNLO: 1% residual theoretical uncertainty).

$gg \to H$ production at the Tevatron and LHC

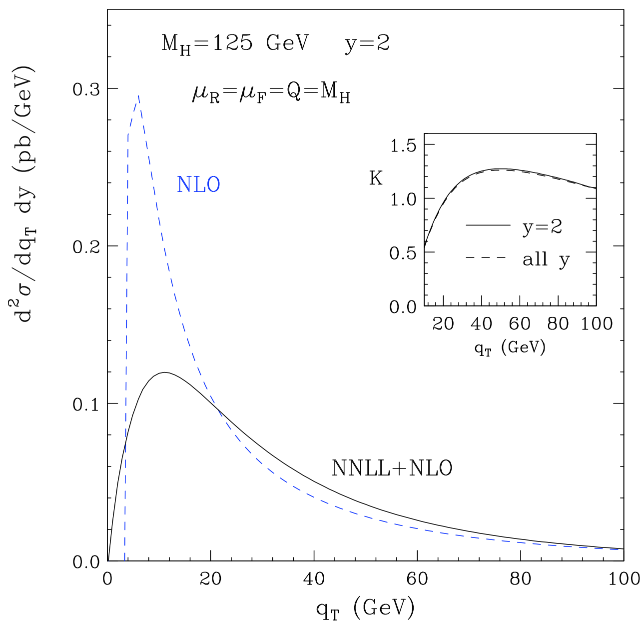
Harlander,Kilgore (03)
Anastasiou,Melnikov,Petriello (03)
Bozzi,Catani,de Florian,Grazzini (04-08)





- dominant production mode in association with $H \to \gamma\gamma$ or $H \to WW$ or $H \to ZZ$;
- dominated by soft dynamics: effective ggH vertex can be used;
- perturbative convergence LO$\to$ NLO$\to$ NNLO.

Inclusive cross section, resum effects of soft radiation:



large $q_T \xrightarrow{q_T > M_H}$

perturbative expansion in $\alpha_s(\mu)$

small $q_T \xrightarrow{q_T \ll M_H}$

need to resum large $\ln(M_H^2/q_T^2)$

Need exclusive NNLO results: e.g. $gg \to H \to \gamma\gamma, WW, ZZ \to$. Extension of (IR safe) subtraction method to NNLO (Catani,Grazzini)

$\Downarrow$

available in HNNLO

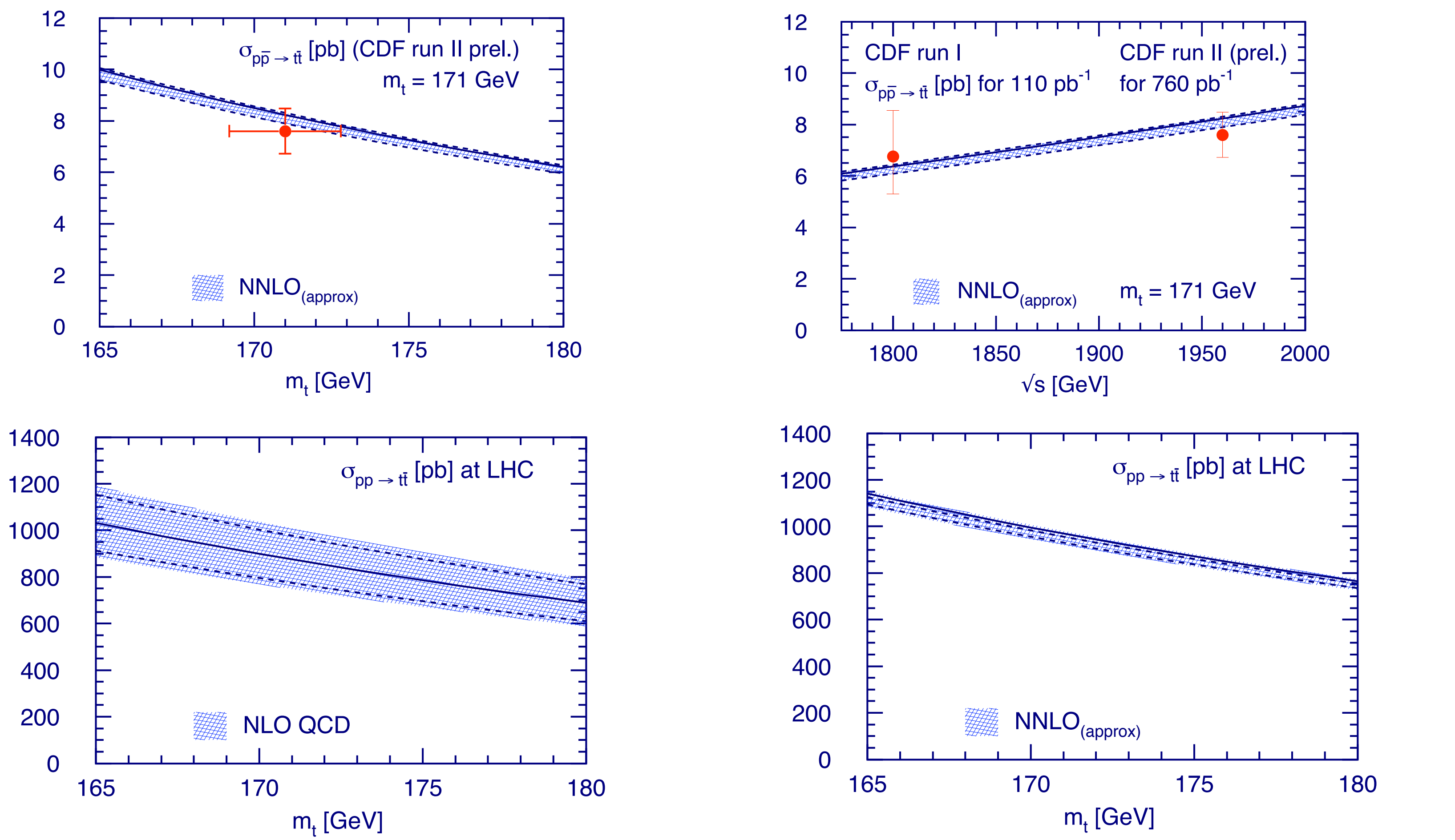
(see also : Anastasiou,Melnikov,Petriello; Binoth,Heinrich)

$Q\bar{Q}$ production at the Tevatron and LHC

Czakon, Mitov, Moch (07-08)

Both $q\bar{q} \to Q\bar{Q}$ and $gg \to Q\bar{Q}$ channels calculated at $O(m_Q^2/s)$ ($s \to$ kinematic scale). Lots of work in progress!

Updated estimate of theoretical precision (from truncated NNLL+NLO calculation of $t\bar{t}$ cross section) (Moch, Uwer):



Conclusions and Outlook

- Enormous theoretical activity in preparing for the LHC: incredible number of crucial NLO and NNLO (!) results already available:
  - NNLO QCD corrections to W/Z production;
  - NLO QCD corrections to $Q\bar{Q}$ production, NNLO QCD corrections are in progress;
  - NLO/NNLO QCD (and EW) corrections to all (to some) Higgs boson production modes;
  - NLO QCD correction to important background modes to Higgs and new physics searches: Z/W+2j, Z/W+2b, WW+j, ...
  - NNLO QCD corrections to hadronic event shapes in e^+e^- annihilations to extract α_s;
  - ...

- Very important issues still open (NLO and higher):
  - interfacing existing NLO (NNLO) parton level results with parton shower Monte Carlo programs;
  - efficient and stable calculation of high multiplicity/higher order processes (typically QCD background processes);
  - automation of such calculations;
  - availability of codes and results.
- Variety of new methods developed to simplify loop calculations, from which we learn that:
  - formal properties of the fundamental objects of QFT can be illuminating: "The revenge of the S-matrix" (L.Dixon)
  - we need to use symmetry properties to reduce the number of object to calculate, to properly connect different pieces of a calculation, to see structures hidden by symmetry-breaking operations (think of color ordered amplitudes, helicity amplitudes, . . ., N=4 SYM, . . .)
- Are new methods generalizable to NNLO, how easily?