

# Higgs Boson Physics, Part II

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## Outline of Part II

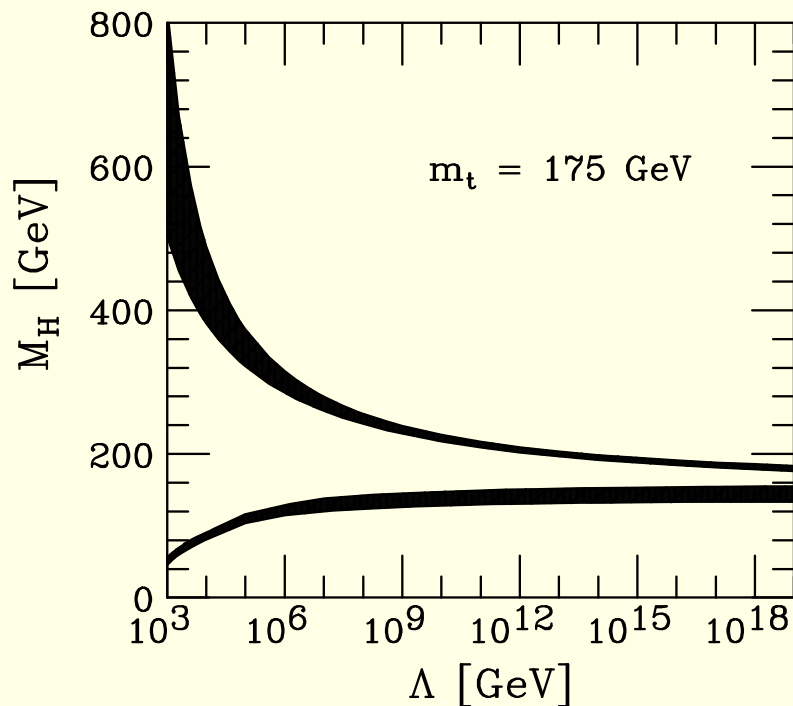
- What do we know about the Standard Model Higgs boson?
  - indirect bounds on  $M_H$  from the theoretical consistency of the Standard Model.
  - indirect bounds from precision fits of electroweak physics observables.
  - direct bounds on  $M_H$  from experimental searches at LEP II.
- The Higgs boson sector of the MSSM
  - General structure.
  - Higgs boson couplings to the SM gauge bosons.
  - Higgs boson couplings to the SM fermions.
  - Higgs boson decay branching ratios.
  - direct bounds on  $M_h$ - $M_A$  from experimental searches at LEP II.

## Some References for Part II

- Theory and Phenomenology of the Higgs boson(s):
  - ▷ The Higgs Hunter Guide,  
J. Gunion, H.E. Haber, G. Kane, and S. Dawson
  - ▷ Introduction to the physics of Higgs bosons,  
S. Dawson, TASI Lectures 1994, hep-ph/9411325
  - ▷ Introduction to electroweak symmetry breaking,  
S. Dawson, hep-ph/9901280
  - ▷ Higgs Boson Theory and Phenomenology,  
M. Carena and H.E. Haber, hep-ph/0208209
- The Minimal Supersymmetric Standard Model:
  - ▷ The Quantum Theory of Fields, V. III, S. Weinberg

# Theoretical constraints on $M_H$ in the Standard Model

SM as an effective theory valid up to a scale  $\Lambda$ . The Higgs sector of the SM actually contains two unknowns:  $M_H$  and  $\Lambda$ .



Bounds given by:

- unitarity
- triviality
- vacuum stability
- EW precision measurements
- fine tuning

$M_H^2 = 2\lambda v^2$  →  $M_H$  determines the weak/strong coupling behavior of the theory, i.e. the limit of validity of the perturbative approach.

Unitarity: longitudinal gauge boson scattering cross section at high energy grows with  $M_H$ .

## Electroweak Equivalence Theorem:

in the high energy limit ( $s \gg M_V^2$ )

$$\mathcal{A}(V_L^1 \dots V_L^n \rightarrow V_L^1 \dots V_L^m) = (i)^n (-i)^m \mathcal{A}(\omega^1 \dots \omega^n \rightarrow \omega^1 \dots \omega^m) + O\left(\frac{M_V^2}{s}\right)$$

( $V_L^i$ =longitudinal weak gauge boson;  $\omega^i$ =associated Goldstone boson).

Example:  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$

$$\mathcal{A}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) \sim -\frac{1}{v^2} \left( -s - t + \frac{s^2}{s - M_H^2} + \frac{t^2}{t - M_H^2} \right)$$

$$\mathcal{A}(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) = -\frac{M_H^2}{v^2} \left( \frac{s}{s - M_H^2} + \frac{t}{t - M_H^2} \right)$$

$\Downarrow$

$$\mathcal{A}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = \mathcal{A}(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) + O\left(\frac{M_W^2}{s}\right)$$

Using partial wave decomposition:

$$\mathcal{A} = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) a_l$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\mathcal{A}^2| \longrightarrow \sigma = \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l+1) |a_l|^2 = \frac{1}{s} \text{Im} [\mathcal{A}(\theta=0)]$$

↓

$$\boxed{|a_l|^2 = \text{Im}(a_l)} \longrightarrow \boxed{|\text{Re}(a_l)| \leq \frac{1}{2}}$$

Most constraining condition for  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  from

$$a_0(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) = -\frac{M_H^2}{16\pi v^2} \left[ 2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s} \log \left( 1 + \frac{s}{M_H^2} \right) \right] \xrightarrow{s \gg M_H^2} -\frac{M_H^2}{8\pi v^2}$$

$$\boxed{|\text{Re}(a_0)| < \frac{1}{2}} \longrightarrow \boxed{M_H < 870 \text{ GeV}}$$

Best constraint from coupled channels ( $2W_L^+ W_L^- + Z_L Z_L$ ):

$$a_0 \xrightarrow{s \gg M_H^2} -\frac{5M_H^2}{32\pi v^2} \longrightarrow \boxed{M_H < 780 \text{ GeV}}$$

Observe that: if there is no Higgs boson, i.e.  $M_H \gg s$ :

$$a_0(\omega^+\omega^- \rightarrow \omega^+\omega^-) \xrightarrow{M_H^2 \gg s} -\frac{s}{32\pi v^2}$$

Imposing the unitarity constraint  $\longrightarrow$   $\sqrt{s_c} < 1.8 \text{ TeV}$

Most restrictive constraint  $\longrightarrow$   $\sqrt{s_c} < 1.2 \text{ TeV}$



New physics expected at the TeV scale

Exciting !!

this is the range of energies of both Tevatron and LHC

**Triviality:** a  $\lambda\phi^4$  theory cannot be perturbative at all scales unless  $\lambda=0$ .

In the SM the **scale evolution of  $\lambda$**  is more complicated:

$$32\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 - (3g'^2 + 9g^2 - 24y_t^2)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2g^2 + \frac{9}{8}g^4 - 24y_t^4 + \dots$$

( $t = \ln(Q^2/Q_0^2)$ ,  $y_t = m_t/v \rightarrow$  top quark Yukawa coupling).

Still, **for large  $\lambda$**  ( $\leftrightarrow$  large  $M_H$ ) the first term dominates and (at 1-loop):

$$\lambda(Q) = \frac{\lambda(Q_0)}{1 - \frac{3}{4\pi^2} \lambda(Q_0) \ln\left(\frac{Q^2}{Q_0^2}\right)}$$

when  $Q$  grows  $\longrightarrow$   $\lambda(Q)$  hits a pole  $\rightarrow$  triviality

**Imposing that  $\lambda(Q)$  is finite,** gives a scale dependent bound on  $M_H$ :

$$\frac{1}{\lambda(\Lambda)} > 0 \longrightarrow M_H^2 < \frac{8\pi^2 v^2}{3 \log\left(\frac{\Lambda^2}{v^2}\right)}$$

where we have set  $Q \rightarrow \Lambda$  and  $Q_0 \rightarrow v$ .



Vacuum stability:  $\lambda(Q) > 0$

For small  $\lambda$  ( $\leftrightarrow$  small  $M_H$ ) the last term in  $d\lambda/dt = \dots$  dominates and:

$$\lambda(\Lambda) = \lambda(v) - \frac{3}{4\pi^2} y_t^2 \log \left( \frac{\Lambda^2}{v^2} \right)$$

from where a first rough lower bound is derived:

$$\lambda(\Lambda) > 0 \longrightarrow M_H^2 > \frac{3v^2}{2\pi^2} y_t^2 \log \left( \frac{\Lambda^2}{v^2} \right)$$

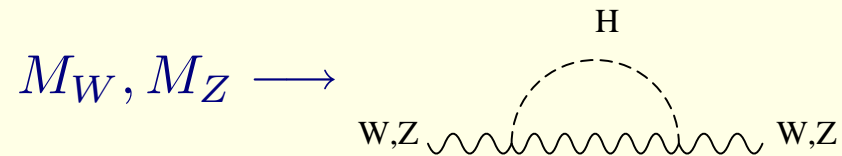
More accurate analyses use 2-loop renormalization group improved  $V_{eff}$ .

EW precision fits: perturbatively calculate observables in terms of few parameters:

$$M_Z, G_F, \alpha(M_Z), M_W, m_f, (\alpha_s(M_Z))$$

extracted from experiments with high accuracy.

- SM needs Higgs boson to cancel infinities, e.g.



- Finite logarithmic contributions survive, e.g. radiative corrections to  $\rho = M_W^2 / (M_Z^2 \cos^2 \theta_W)$ :

$$\rho = 1 - \frac{11g^2}{96\pi^2 \tan^2 \theta_W} \ln \left( \frac{M_H}{M_W} \right)$$

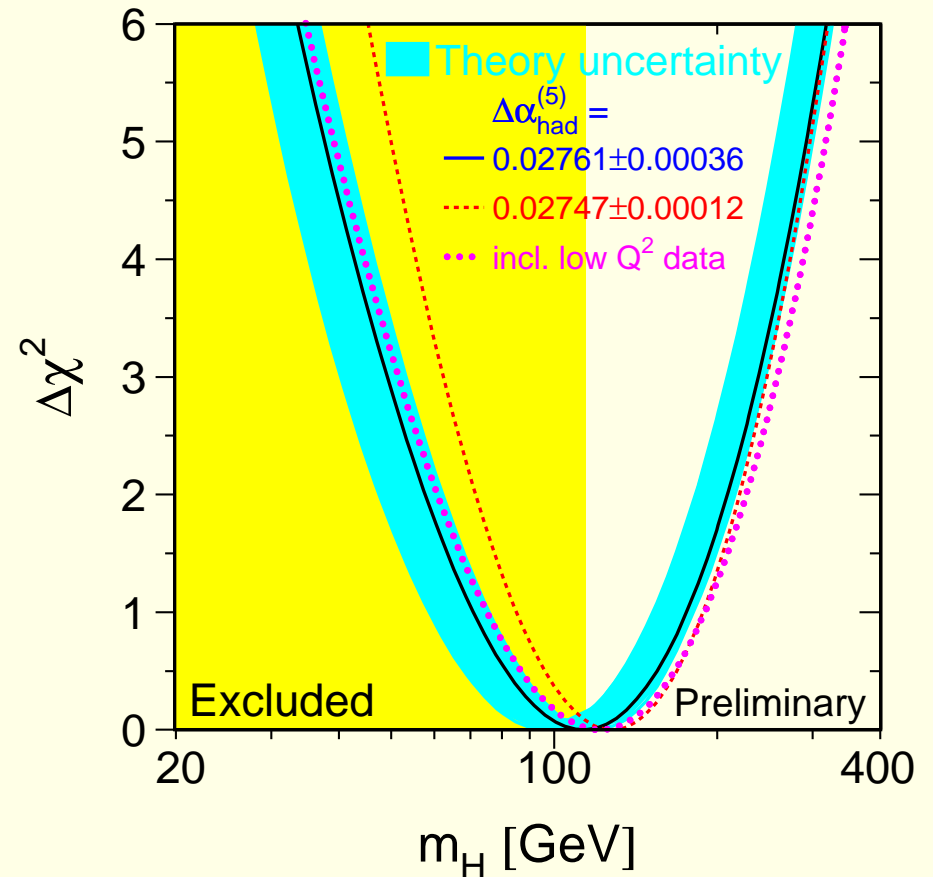
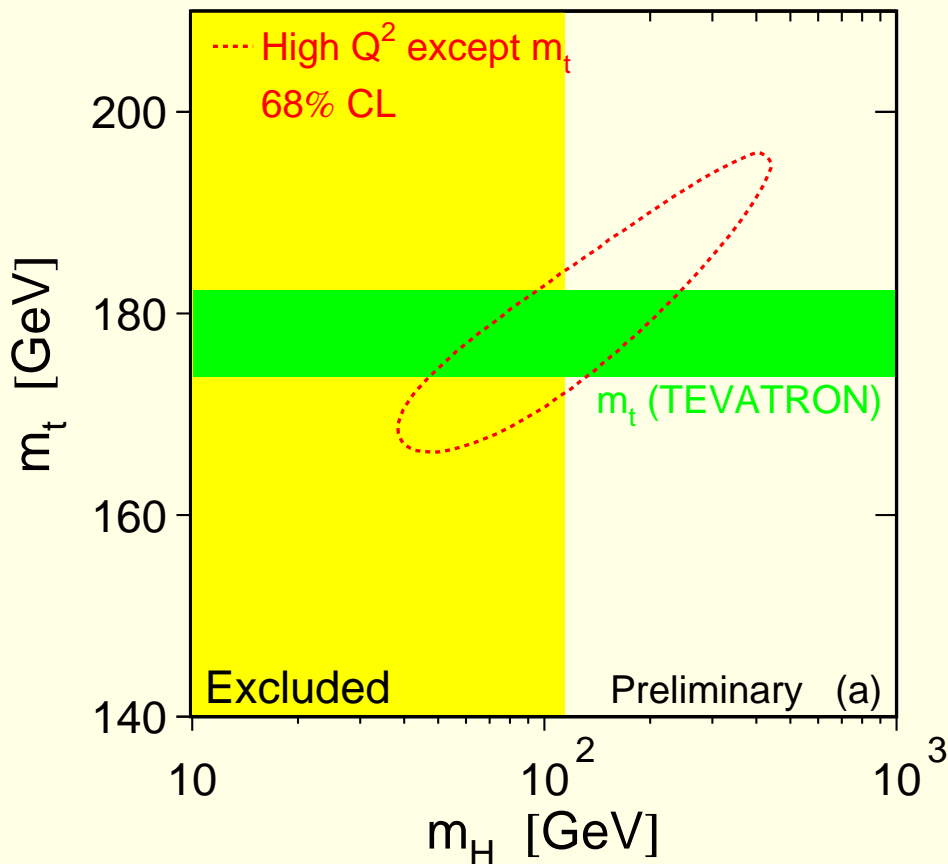
Main effects in oblique radiative corrections (S,T-parameters)

- New physics at the scale  $\Lambda$  will appear as higher dimension effective operators.

$M_H$  only unknown input...

Precision measurements of the SM, which fully test the quantum structure of the theory by including higher order loop corrections, constrain the mass of the SM Higgs to be light:

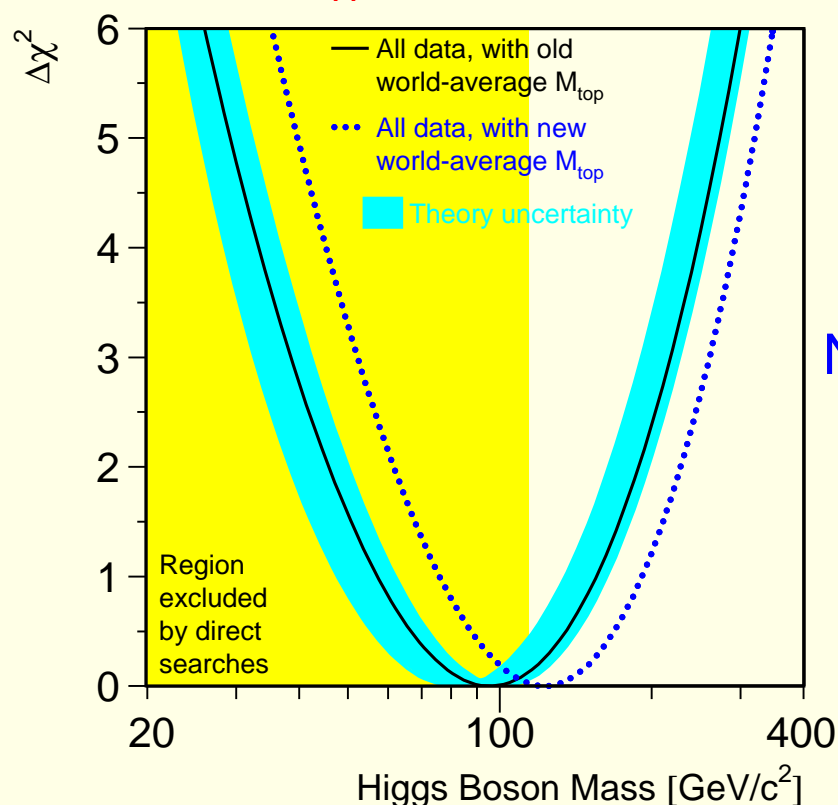
$$M_H < 251 \text{ GeV} \quad (95\% \text{cl})$$



# Consequences for Standard-Model Higgs

Precision electroweak data:

Constrain  $M_H$  in the MSM



Old:

$$M_{top} = 174.3 \pm 5.1 \text{ GeV}$$

$$\log M_H = 1.98^{+0.21}_{-0.22}$$

$$M_H = 96^{+60}_{-38} \text{ GeV}$$

$$\text{or } < 219 \text{ GeV (95\% CL)}$$

New:

$$M_{top} = 178.0 \pm 4.3 \text{ GeV}$$

$$\log M_H = 2.07^{+0.20}_{-0.21}$$

$$M_H = 117^{+67}_{-45} \text{ GeV}$$

$$\text{or } < 251 \text{ GeV (95\% CL)}$$

(Procedure as in hep-ex/0312023!)

MWG

(from the EWWG home page: [lepewwg.web.cern.ch/LEPEWWG/](http://lepewwg.web.cern.ch/LEPEWWG/))

Fine-tuning:  $M_H$  is unstable to ultraviolet corrections

$$M_H^2 = (M_H^0)^2 + \frac{g^2}{16\pi^2} \Lambda^2 \cdot \text{constant} + \text{higher orders}$$

$M_H^0 \rightarrow$  fundamental parameter of the SM

$\Lambda \rightarrow$  UV-cutoff scale

Unless  $\Lambda \simeq$  EW-scale, fine-tuning is required to get  $M_H \simeq$  EW-scale.

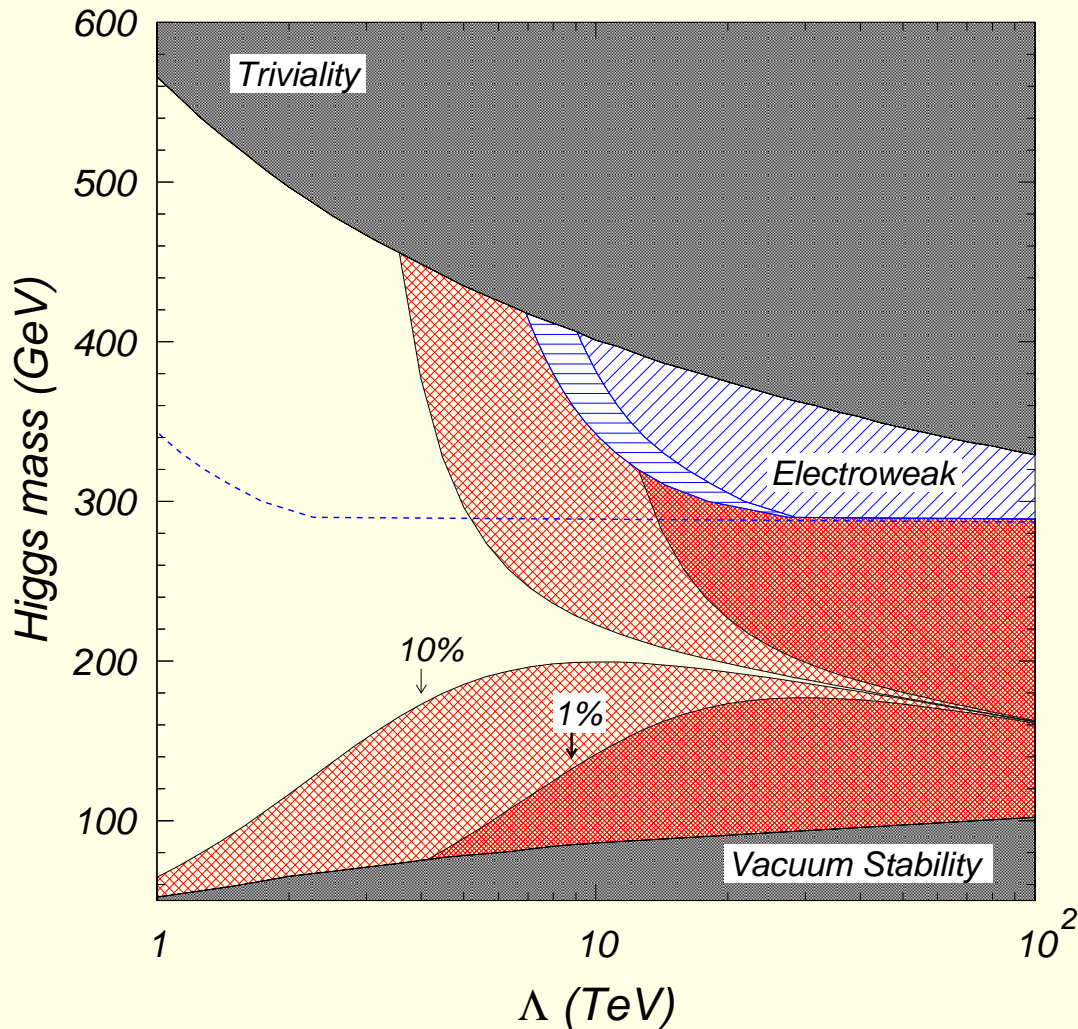
More generally, the all order calculation of  $V_{eff}$  would give:

$$\bar{\mu}^2 = \mu^2 + \Lambda^2 \sum_{n=0}^{\infty} c_n(\lambda_i) \log^n(\Lambda/Q)$$

Veltmann condition: the absence of large quadratic corrections is guaranteed by:

$$\sum_{n=0}^{\infty} c_n(\lambda_i) \log^n(\Lambda/M_H) = 0 \quad \text{or better} \quad \sum_{n=0}^{n_{max}} c_n(\lambda_i) \log^n(\Lambda/M_H) = 0$$

In summary:



amount of fine tuning =

$$\frac{2\Lambda^2}{M_H^2} \left| \sum_{n=0}^{n_{max}} c_n(\lambda_i) \log^n(\Lambda/M_H) \right|$$

←  $n_{max} = 1$

(C. Kolda and H. Murayama, hep-ph/0003170)

Adding more Higgs fields ... Main constraints from:

- $\rho$  parameter:  $M_W^2 = \rho M_Z^2 \cos^2 \theta_W$ ,  $\rho \simeq 1$ .
- flavor changing neutral currents.

Ex.: Two Higgs Doublet Models (2HDM)

$$\mathcal{L}_{Yukawa} = -\Gamma_{ij,k}^u \bar{Q}_L^i \Phi^{k,c} u_R^j + -\Gamma_{ij,k}^d \bar{Q}_L^i \Phi^k d_R^j + \text{h.c.} \quad \text{for } k = 1, 2$$

$$\Downarrow \quad \langle \Phi^k \rangle = \frac{v^k}{\sqrt{2}} \longrightarrow \Phi^k = \Phi'^k + v^k$$

$$\mathcal{L}_{Yukawa} = -\bar{u}_L^i \underbrace{\sum_k \Gamma_{ij,k}^u \frac{v^k}{\sqrt{2}}}_{M_{ij}^u} u_R^j - \bar{d}_L^i \underbrace{\sum_k \Gamma_{ij,k}^d \frac{v^k}{\sqrt{2}}}_{M_{ij}^d} d_R^j + \text{h.c.} + \text{FC couplings}$$

Avoid FC couplings by imposing **ad hoc discrete symmetry**:

$$\left\{ \begin{array}{l} \Phi^1 \rightarrow -\Phi^1 \quad \text{and} \quad \Phi^2 \rightarrow \Phi^2 \\ d^i \rightarrow -d^i \quad \text{and} \quad u^j \rightarrow \pm u^j \end{array} \right. \longrightarrow \left\{ \begin{array}{l} \text{Model I: } u \text{ and } d \text{ coupled to same doublet} \\ \text{Model II: } u \text{ and } d \text{ coupled to different doublets} \end{array} \right.$$

# The Higgs bosons of the MSSM

Two complex  $SU(2)_L$  doublets, with hypercharge  $Y = \pm 1$ :

$$\Phi_u = \begin{pmatrix} \phi_u^+ \\ \phi_u^0 \end{pmatrix}, \quad \Phi_d = \begin{pmatrix} \phi_d^0 \\ \phi_d^- \end{pmatrix}$$

and (super)potential (Higgs part only):

$$\begin{aligned} V_H &= (|\mu|^2 + m_u^2)|\Phi_u|^2 + (|\mu|^2 + m_d^2)|\Phi_d|^2 - \mu B \epsilon_{ij} (\Phi_u^i \Phi_d^j + h.c.) \\ &+ \frac{g^2 + g'^2}{8} (|\Phi_u|^2 - |\Phi_d|^2)^2 + \frac{g^2}{2} |\Phi_u^\dagger \Phi_d|^2 \end{aligned}$$

The EW symmetry is spontaneously broken by choosing:

$$\langle \Phi_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad \langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix}$$

normalized to preserve the SM relation:

$$M_W^2 = g^2(v_u^2 + v_d^2)/4 = g^2 v^2/4.$$



Five physical scalar/pseudoscalar degrees of freedom:

$$h^0 = -(\sqrt{2}\text{Re}\Phi_d^0 - v_d) \sin \alpha + (\sqrt{2}\text{Re}\Phi_u^0 - v_u) \cos \alpha$$

$$H^0 = (\sqrt{2}\text{Re}\Phi_d^0 - v_d) \cos \alpha + (\sqrt{2}\text{Re}\Phi_u^0 - v_u) \sin \alpha$$

$$A^0 = \sqrt{2} (\text{Im}\Phi_d^0 \sin \beta + \text{Im}\Phi_u^0 \cos \beta)$$

$$H^\pm = \Phi_d^\pm \sin \beta + \Phi_u^\pm \cos \beta$$

where  $\boxed{\tan \beta = v_u/v_d}$ .

All masses can be expressed (at tree level) in terms of  $\boxed{\tan \beta}$  and  $M_A$ :

$$M_{H^\pm}^2 = M_A^2 + M_W^2$$

$$M_{H,h}^2 = \frac{1}{2} \left( M_A^2 + M_Z^2 \pm ((M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta)^{1/2} \right)$$

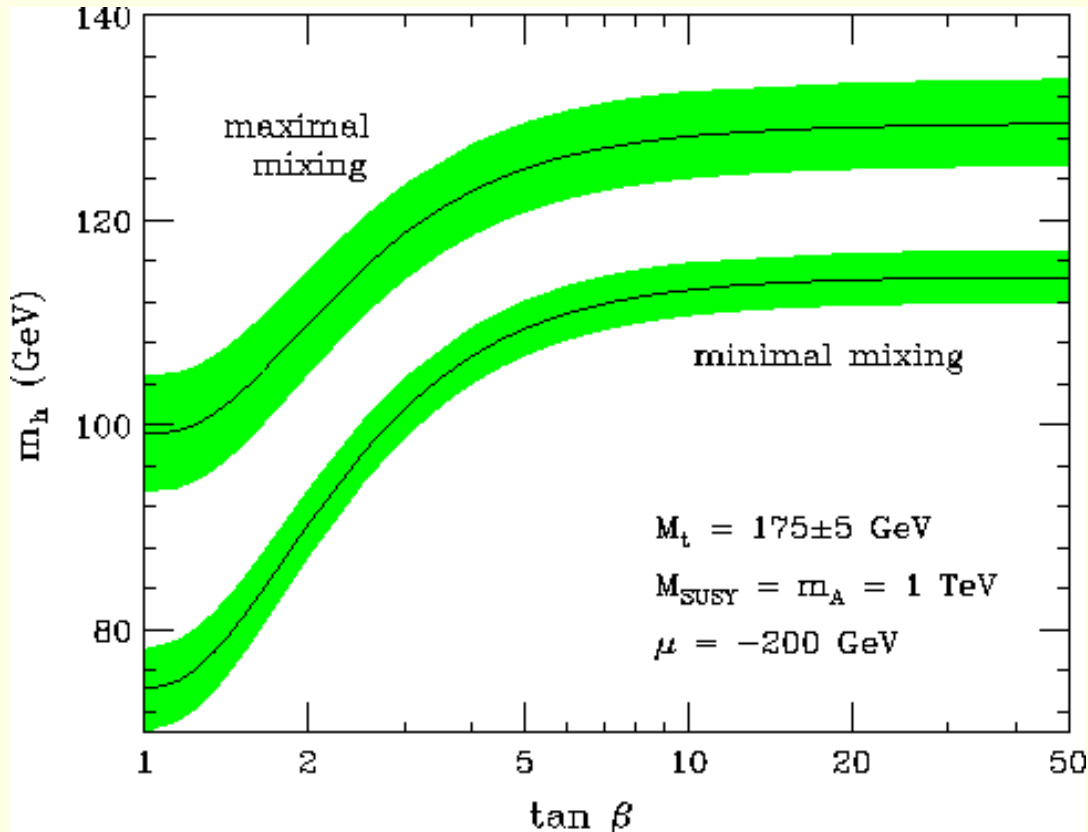
**Notice:** tree level upper bound on  $M_h$ :  $\boxed{M_h^2 \leq M_Z^2 \cos 2\beta \leq M_Z^2}$  !

Higgs masses greatly modified by radiative corrections.

In particular, the upper bound on  $M_h$  becomes:

$$M_h^2 \leq M_Z^2 + \frac{3g^2 m_t^2}{8\pi^2 M_W^2} \left[ \log \left( \frac{M_S^2}{m_t^2} \right) + \frac{X_t^2}{M_S^2} \left( 1 - \frac{X_t^2}{12M_S^2} \right) \right]$$

where  $M_S \equiv (M_{\tilde{t}_1}^2 + M_{\tilde{t}_2}^2)/2$  while  $X_t$  is the top squark mixing parameter:



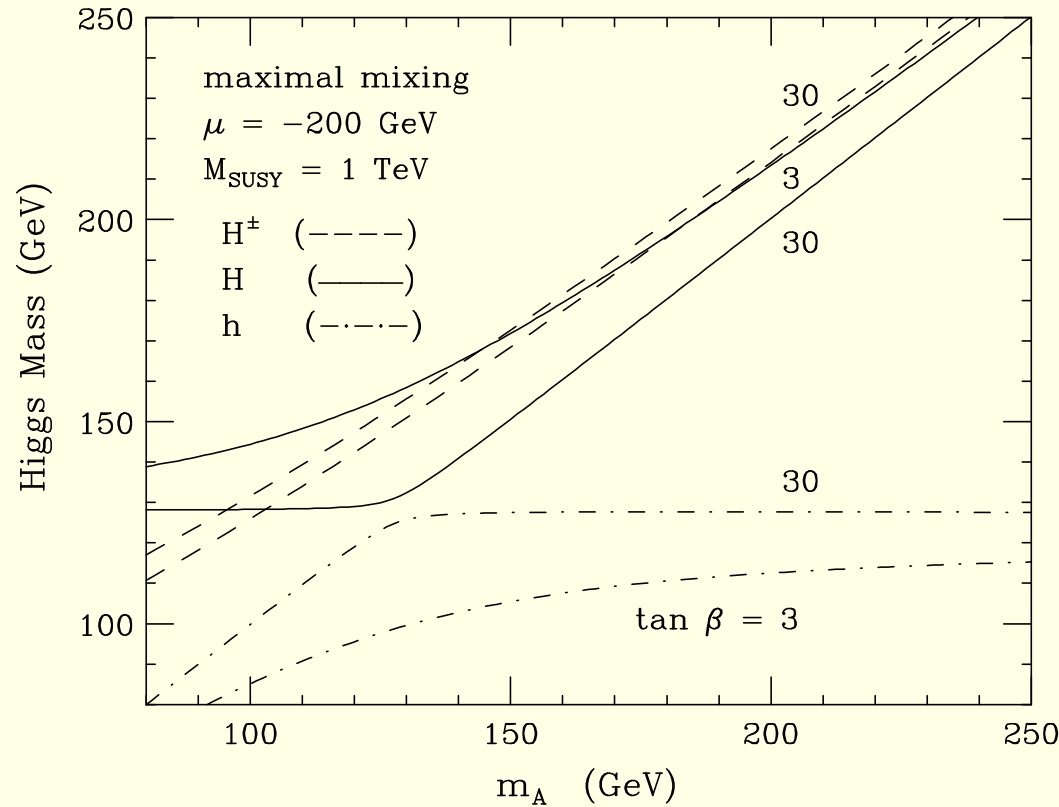
$$\begin{pmatrix} M_{Q_t}^2 + m_t^2 + D_L^t & m_t X_t \\ m_t X_t & M_{R_t}^2 + m_t^2 + D_R^t \end{pmatrix}$$

with  $X_t \equiv A_t - \mu \cot \beta$ .

$$D_L^t = (1/2 - 2/3 \sin^2 \theta_W) M_Z^2 \cos 2\beta$$

$$D_R^t = 2/3 \sin^2 \theta_W M_Z^2 \cos 2\beta$$

In summary:



$$\boxed{M_h^{max} \simeq 135 \text{ GeV}} \quad (\text{if top-squark mixing is maximal})$$

Moreover, interesting “sum-rule”:

$$M_H^2 \cos^2(\beta - \alpha) + M_h^2 \sin^2(\beta - \alpha) = [M_h^{max} \tan \beta]^2$$

$$\text{large } \tan \beta \longrightarrow \begin{cases} M_A > M_h^{max} \longrightarrow M_h \simeq M_h^{max}, M_H \simeq M_A \\ M_A < M_h^{max} \longrightarrow M_H \simeq M_h^{max}, M_h \simeq M_A \end{cases}$$

Higgs boson couplings to SM gauge bosons:

Some phenomenologically important ones:

$$g_{hVV} = g_V M_V \sin(\beta - \alpha) g^{\mu\nu} \quad , \quad g_{HVV} = g_V M_V \cos(\beta - \alpha) g^{\mu\nu}$$

where  $g_V = 2M_V/v$  for  $V = W, Z$ , and

$$g_{hAZ} = \frac{g \cos(\beta - \alpha)}{2 \cos \theta_W} (p_h - p_A)^\mu \quad , \quad g_{HAZ} = -\frac{g \sin(\beta - \alpha)}{2 \cos \theta_W} (p_H - p_A)^\mu$$

**Notice:**  $\boxed{g_{AZZ} = g_{AWW} = 0}$  ,  $\boxed{g_{H^\pm ZZ} = g_{H^\pm WW} = 0}$

**Decoupling limit:**  $\boxed{M_A \gg M_Z}$   $\longrightarrow$   $\left\{ \begin{array}{l} M_h \simeq M_h^{max} \\ M_H \simeq M_{H^\pm} \simeq M_A \end{array} \right.$

$$\cos^2(\beta - \alpha) \simeq \frac{M_Z^4 \sin^2 4\beta}{M_A^4} \longrightarrow \left\{ \begin{array}{l} \cos(\beta - \alpha) \rightarrow 0 \\ \sin(\beta - \alpha) \rightarrow 1 \end{array} \right.$$

The only low energy Higgs is  $h \simeq H_{SM}$ .

## Higgs boson couplings to quarks and leptons:

Yukawa type couplings,  $\Phi_u$  to up-component and  $\Phi_d$  to down-component of  $SU(2)_L$  fermion doublets. Ex. (3<sup>rd</sup> generation quarks):

$$\mathcal{L}_{Yukawa} = h_t [\bar{t}P_L t \Phi_u^0 - \bar{t}P_L b \Phi_u^+] + h_b [\bar{b}P_L b \Phi_d^0 - \bar{b}P_L t \Phi_d^-] + \text{h.c.}$$

and similarly for leptons. The corresponding couplings can be expressed as ( $y_t, y_b \rightarrow \text{SM}$ ):

$$\begin{aligned} g_{ht\bar{t}} &= \frac{\cos \alpha}{\sin \beta} y_t = [\sin(\beta - \alpha) + \cot \beta \cos(\beta - \alpha)] y_t \\ g_{hb\bar{b}} &= -\frac{\sin \alpha}{\cos \beta} y_b = [\sin(\beta - \alpha) - \tan \beta \cos(\beta - \alpha)] y_b \\ g_{Ht\bar{t}} &= \frac{\sin \alpha}{\sin \beta} y_t = [\cos(\beta - \alpha) - \cot \beta \sin(\beta - \alpha)] y_t \\ g_{Hb\bar{b}} &= \frac{\cos \alpha}{\cos \beta} y_b = [\cos(\beta - \alpha) + \tan \beta \sin(\beta - \alpha)] y_b \\ g_{At\bar{t}} &= \cot \beta \quad , \quad g_{Ab\bar{b}} = \tan \beta \\ g_{H^\pm t\bar{b}} &= \frac{g}{2\sqrt{2}M_W} [m_t \cot \beta (1 - \gamma_5) + m_b \tan \beta (1 + \gamma_5)] \end{aligned}$$

**Notice:** consistent decoupling limit behavior.

# Higgs couplings modified by radiative corrections

Most important effects:

- Corrections to  $\cos(\beta - \alpha)$ : crucial in decoupling behavior.

$$\cos(\beta - \alpha) = K \left[ \frac{M_Z^2 \sin 4\beta}{2M_A^2} + \mathcal{O}\left(\frac{M_Z^4}{M_A^4}\right) \right]$$

where

$$K \equiv 1 + \frac{\delta\mathcal{M}_{11}^2 - \delta\mathcal{M}_{22}^2}{2M_Z^2 \cos 2\beta} - \frac{\delta\mathcal{M}_{12}^2}{M_Z^2 \sin 2\beta}$$

$\delta\mathcal{M}_{ij} \longrightarrow$  corrections to the CP-even scalar mass matrix.

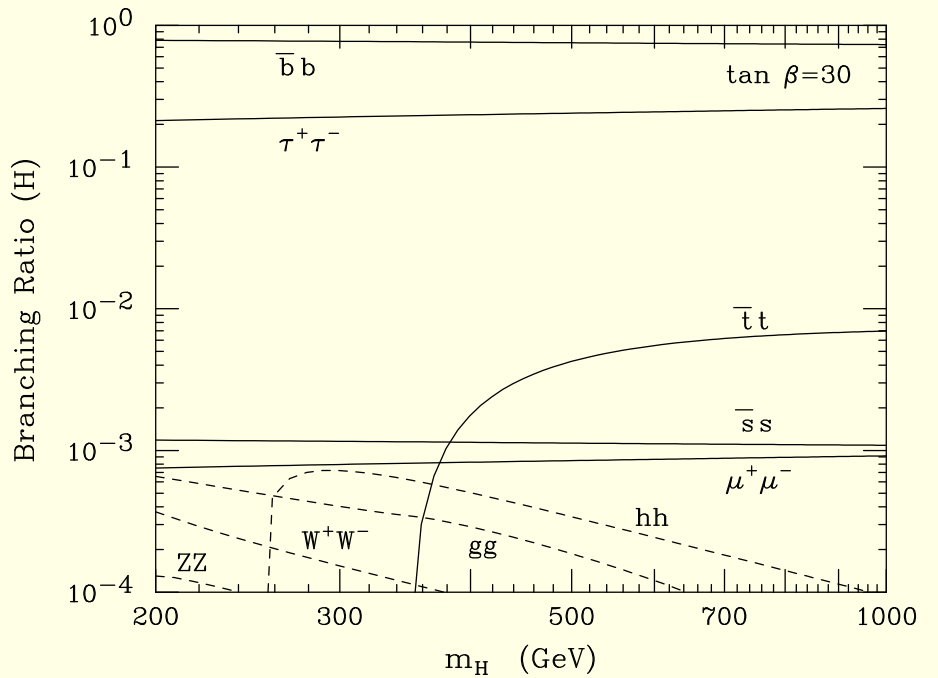
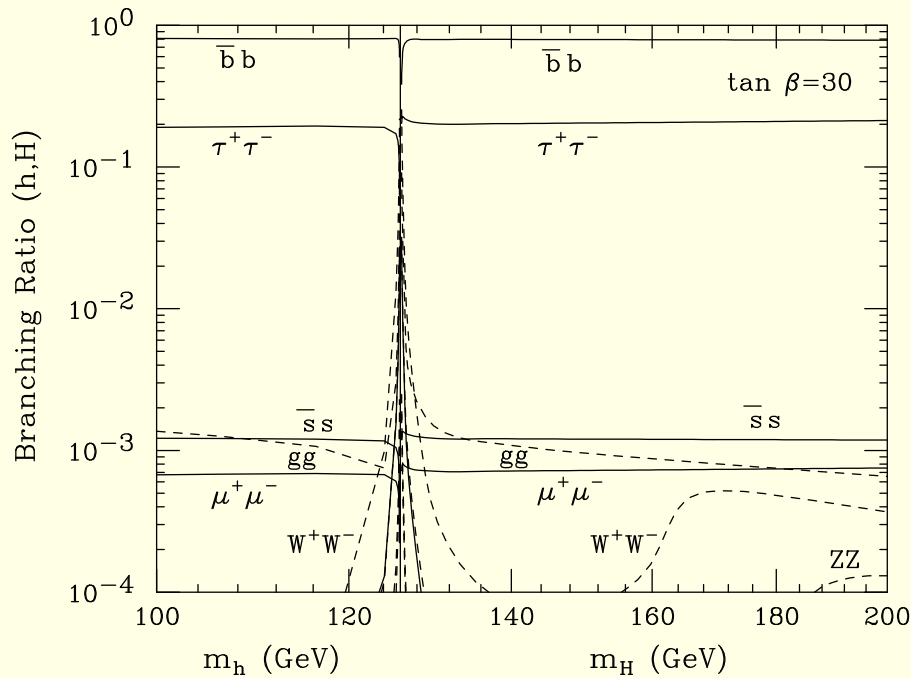
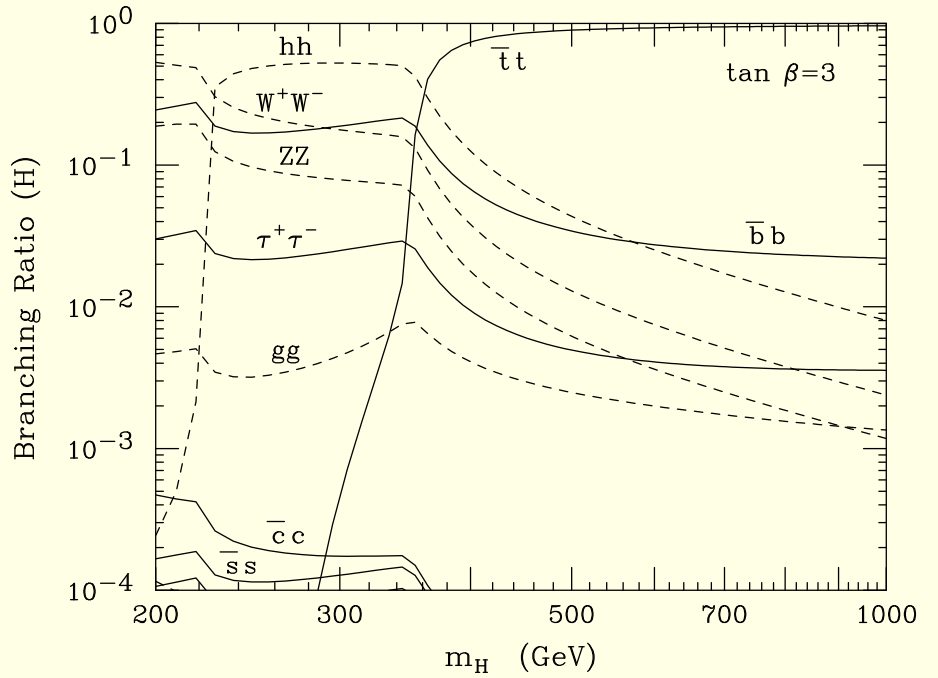
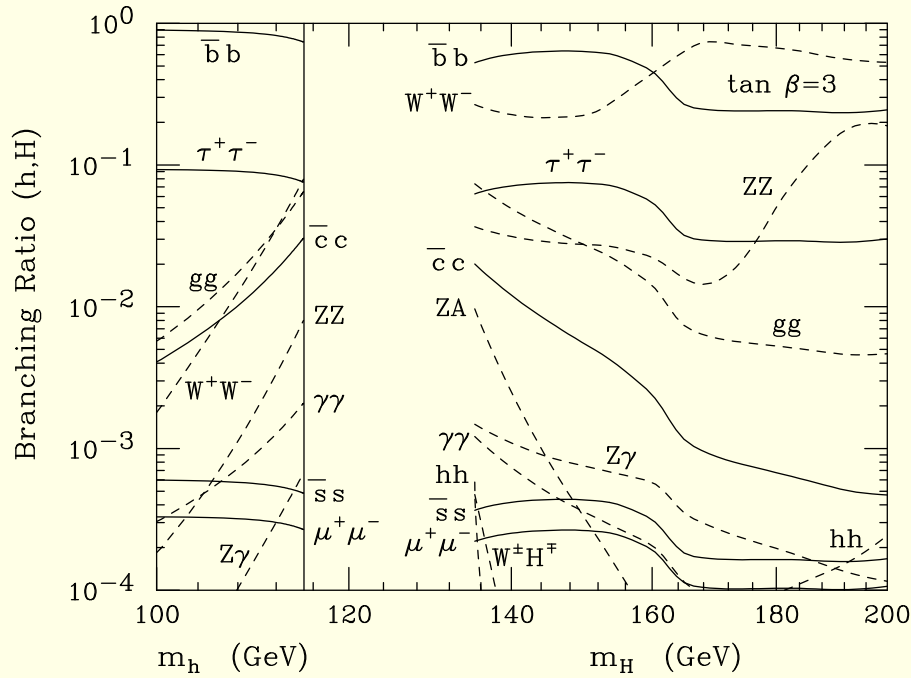
- Corrections to 3<sup>rd</sup> generation Higgs-fermion Yukawa couplings.

$$-\mathcal{L}_{eff} = \epsilon_{ij} \left[ (h_b + \delta h_b) \bar{b}_R H_d^i Q_L^j + (h_t + \delta h_t) \bar{t}_R H_u^j Q_L^i \right] + \Delta h_t \bar{t}_R Q_L^k H_d^{k*} + \Delta h_b \bar{b}_R Q_L^k H_u^{k*} + \text{h.c.}$$

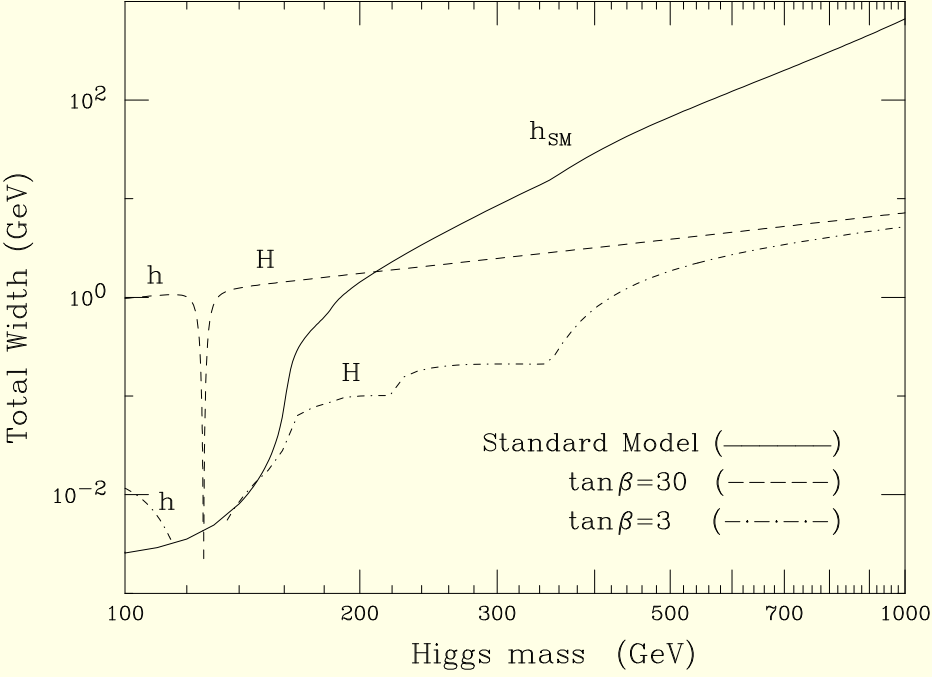
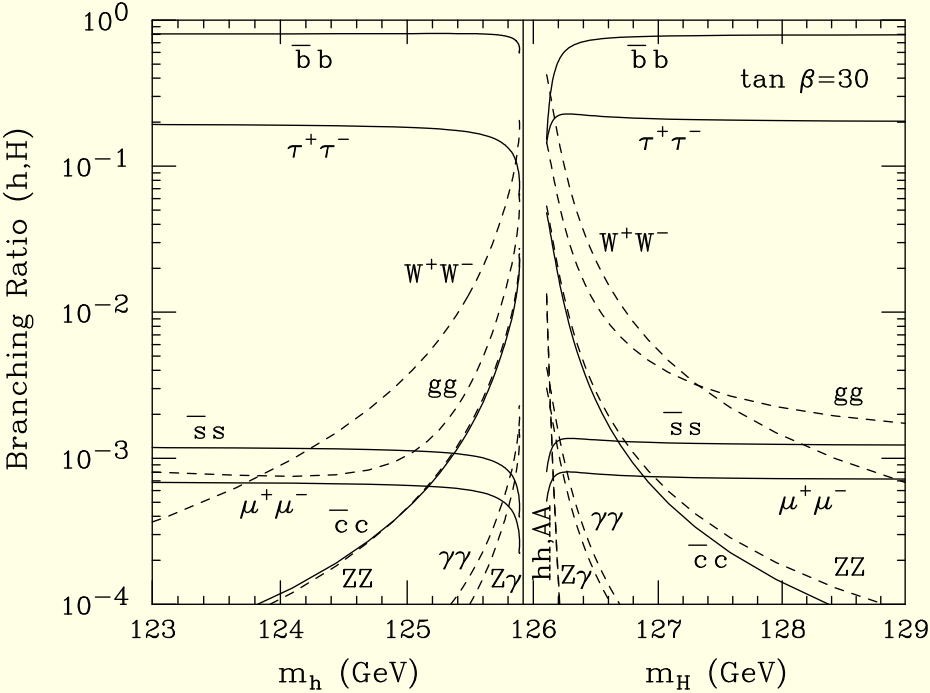
$$m_b = \frac{h_b v}{\sqrt{2}} \cos \beta \left( 1 + \frac{\delta h_b}{h_b} + \frac{\Delta h_b \tan \beta}{h_b} \right) \equiv \frac{h_b v}{\sqrt{2}} \cos \beta (1 + \Delta_b)$$

$$m_t = \frac{h_t v}{\sqrt{2}} \sin \beta \left( 1 + \frac{\delta h_t}{h_t} + \frac{\Delta h_t \tan \beta}{h_t} \right) \equiv \frac{h_t v}{\sqrt{2}} \sin \beta (1 + \Delta_t)$$

# MSSM Higgs boson branching ratios, possible scenarios:

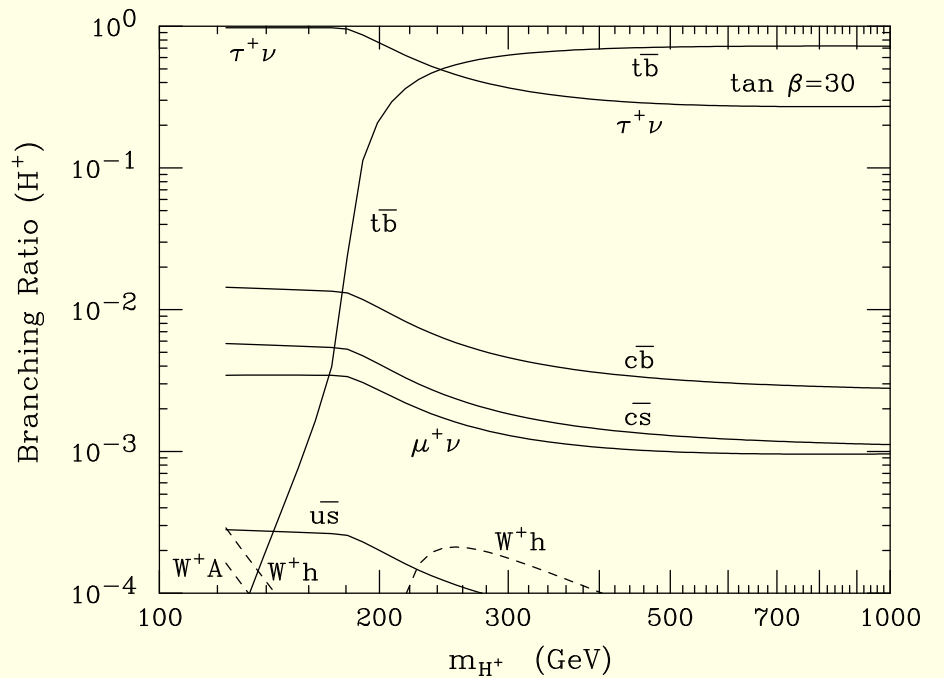
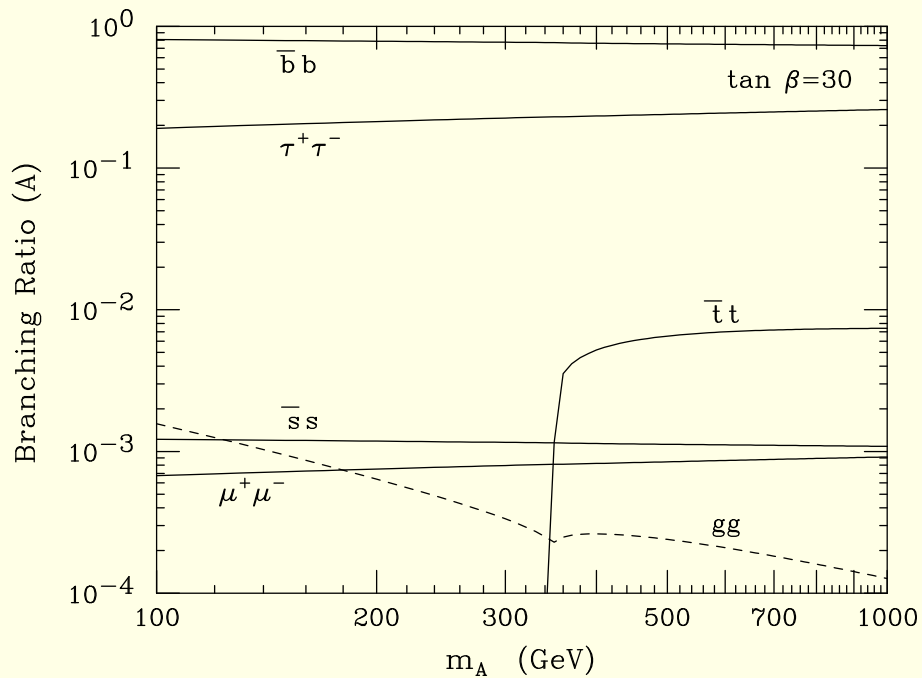
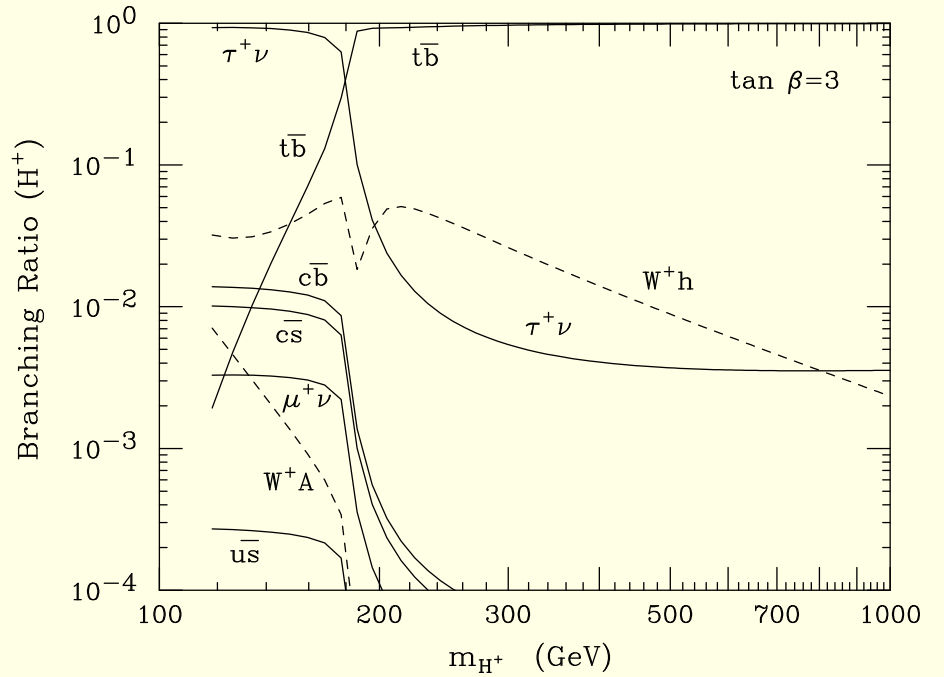
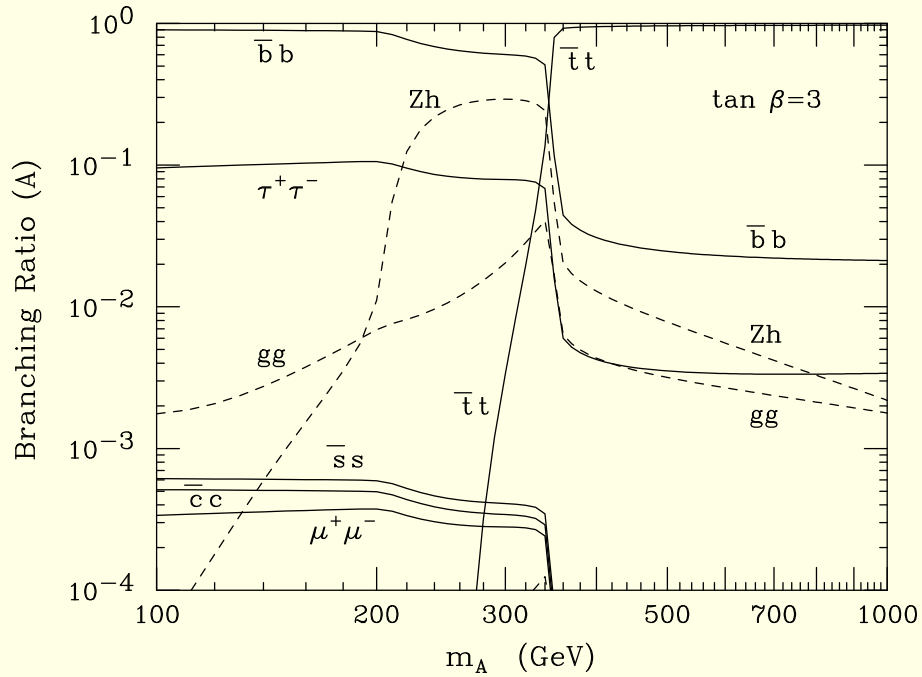


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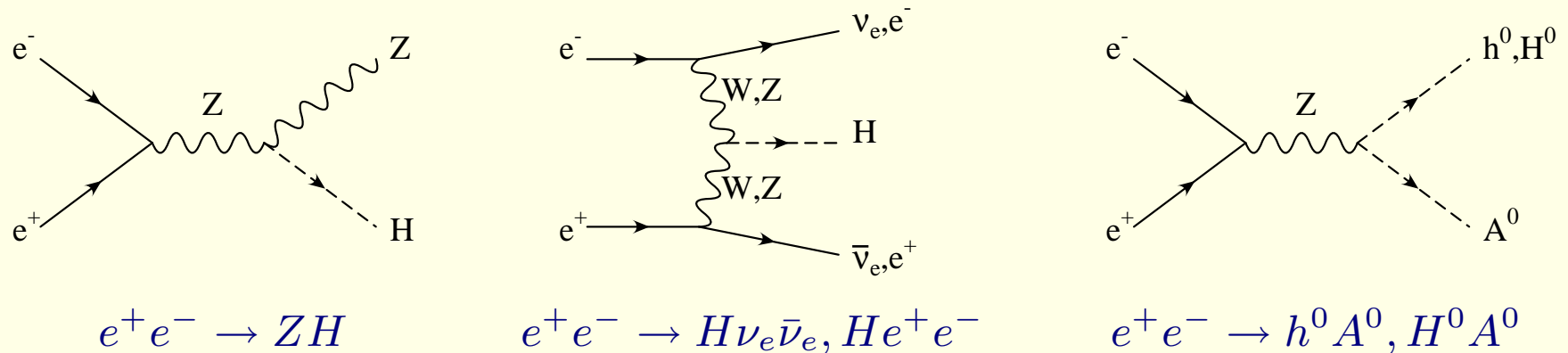


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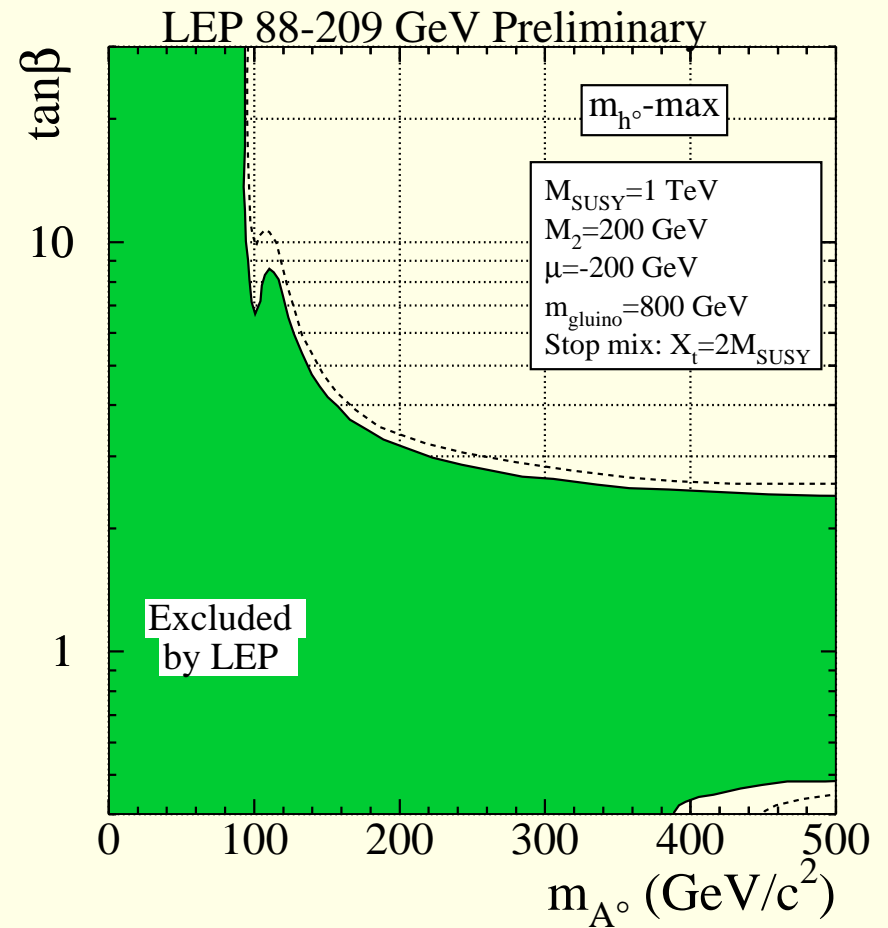
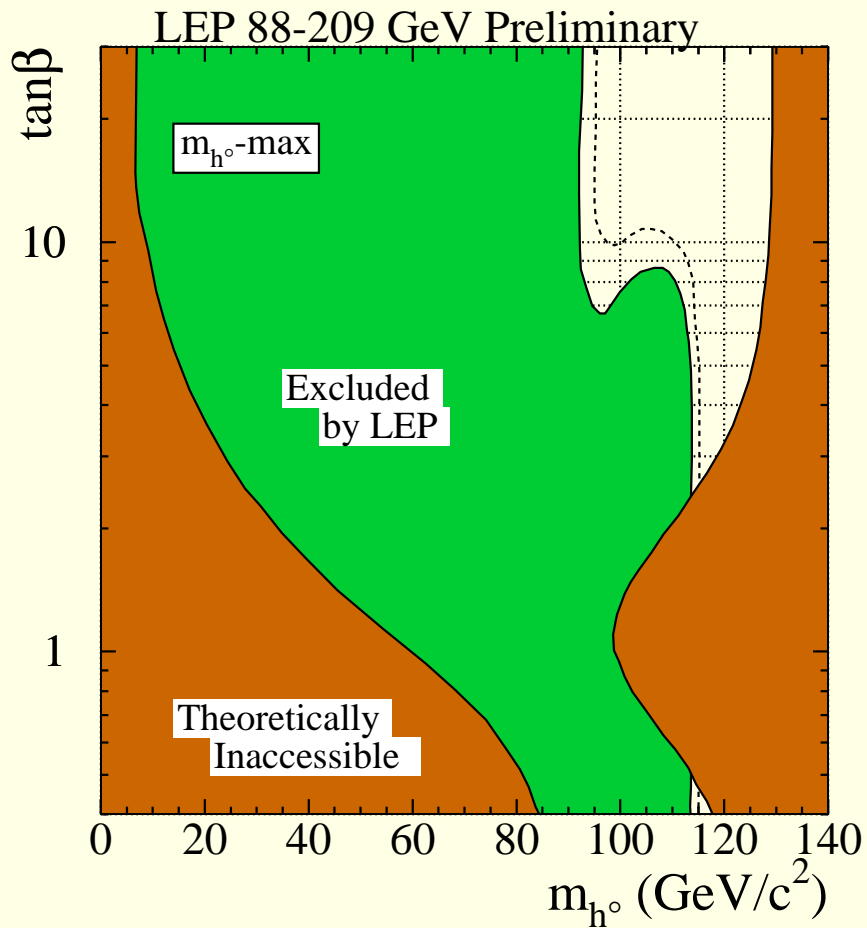
# Experimental searches for a Higgs boson

LEP looked for the SM and the MSSM Higgs bosons in three channels:



The absence of a convincing signal sets the lower bounds:

$$\sqrt{s_{\max}} = 189 - 209 \text{ GeV} \begin{cases} M_H > 114.4 \text{ GeV} & (95\% \text{ c.l.}) \\ M_{h^0/H^0} > 91.0 \text{ GeV}, M_{A^0} > 91.9 \text{ GeV} & (95\% \text{ c.l.}) \end{cases}$$

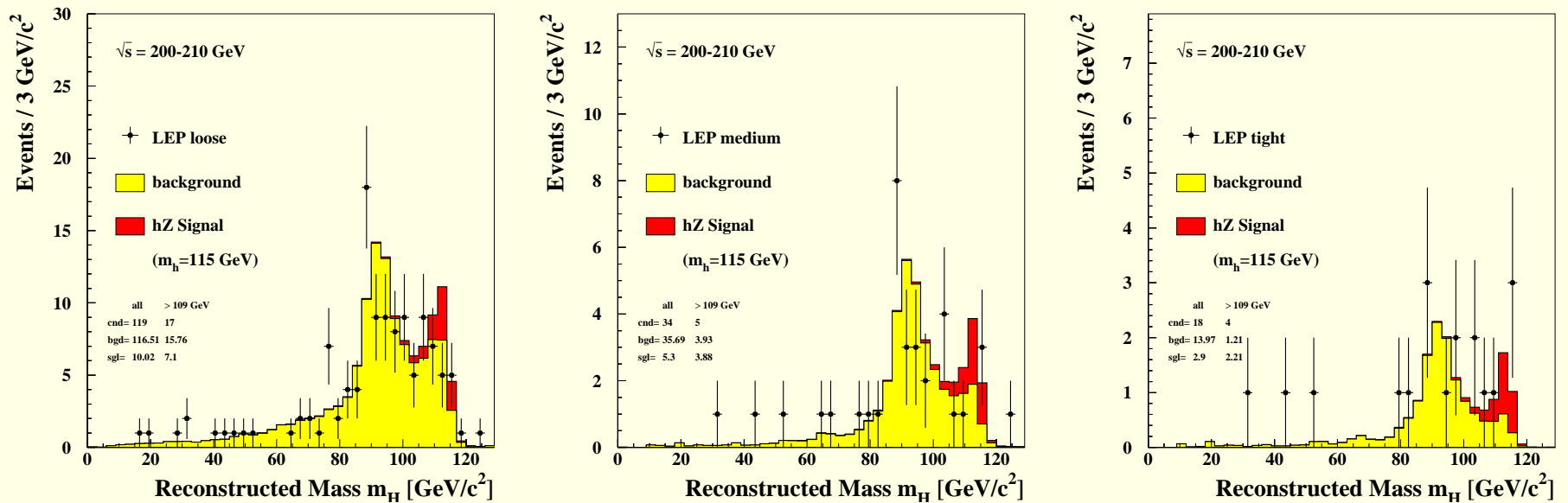


SM Higgs  $\longrightarrow$  CERN-EP/2003-011

MSSM Higgs  $\longrightarrow$  LHWG Note 2001-04

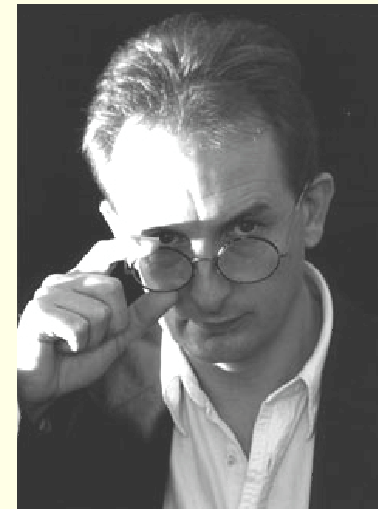
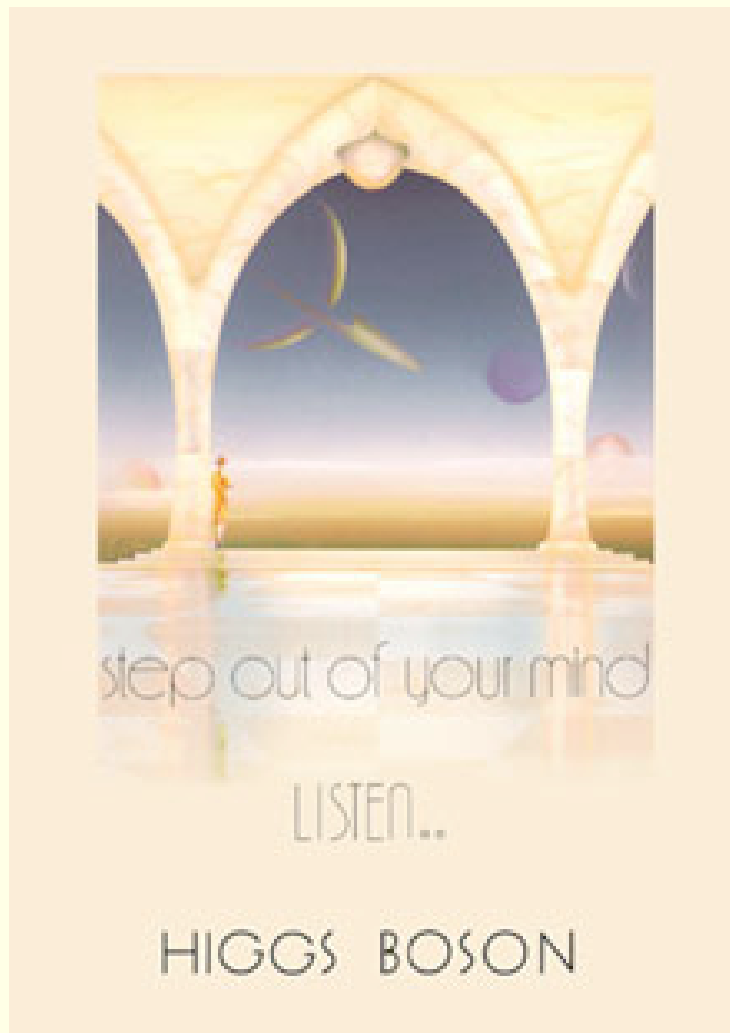
LHWG home page: <http://lephiggs.web.cern.ch/LEPHIGGS/www>

# Some puzzling events around $M_H = 115$ GeV ...



“The resolution of this puzzle is now left to the Fermilab’s Tevatron and the LHC.” (L. Maiani, CERN Director)

# Searching for (the) Higgs Boson ...



... a jazz fusion pianist in the UK ...

## Searching for (the) Higgs Boson ...



“... The Higgs Boson wheel isn't really a wheel. It isn't even designed to roll on; it's designed to grind with. As any anti-rocker skater knows...”

“ ... Anti-rocker skating isn't for everyone, and the Higgs Boson certainly isn't for everyone. But we love it. And we feel it satisfies a need that has been ignored for too long...”