$\sigma_{I\eta I\eta}$

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SHOWER WIDTH

- As I have doubtless said previously, photon identification is difficult. There are many reasons for this, but foremost among them is that you have few direct measurements related to the object itself.
  - We just did an exercise on the clustering, so that's part of it. Another part related to that would be the best estimate of the energy of the photon.
  - But how the energy is distributed within the shower is key to discriminating against hadronic jets. Since jets are typically composed of more particles, the distribution of energy tends to be broader than that of a single electromagnetic shower.
  - Thus "Shower Width".

WHAT IS THIS THING?

- Lots of people know the name, but few know what the calculation is, and why the calculation looks the way it does.
- You have two equations:

$$\sigma^2_{i\eta i\eta} = \frac{\sum_{5x5} w_i(\bar{\eta} - \eta_i)^2}{\sum_{5x5} w_i};$$

$$w_i = \max(0., w_0 + \ln(\frac{E_i}{E_{5x5}}))$$

If it is blithely obvious to you why these two equations are used, then go back to checking your email.

THE FIRST EQUATION:

- Concentrate on the first equation for a moment:
  - Does it look familiar?

$$\sigma^2_{i\eta i\eta} = \frac{\sum_{5x5} w_i(\bar{\eta}-\eta_i)^2}{\sum_{5x5} w_i} = \frac{\sum_{5x5} \bar{\eta}^2 w_i}{\sum_{5x5} w_i} - \frac{2\sum_{5x5} \bar{\eta}\eta_i w_i}{\sum_{5x5} w_i} + \frac{\sum_{5x5} \eta_i^2 w_i}{\sum_{5x5} w_i}$$

$$= \bar{\eta}^2 - \frac{2\bar{\eta}\sum_{5x5} \eta_i w_i}{\sum_{5x5} w_i} + \frac{\sum_{5x5} \eta_i^2 w_i}{\sum_{5x5} w_i}; \bar{\eta} = <\eta> = \frac{\sum_{5x5} \eta_i w_i}{\sum_{5x5} w_i}; <\eta^2> = \frac{\sum_{5x5} \eta_i^2 w_i}{\sum_{5x5} w_i};$$

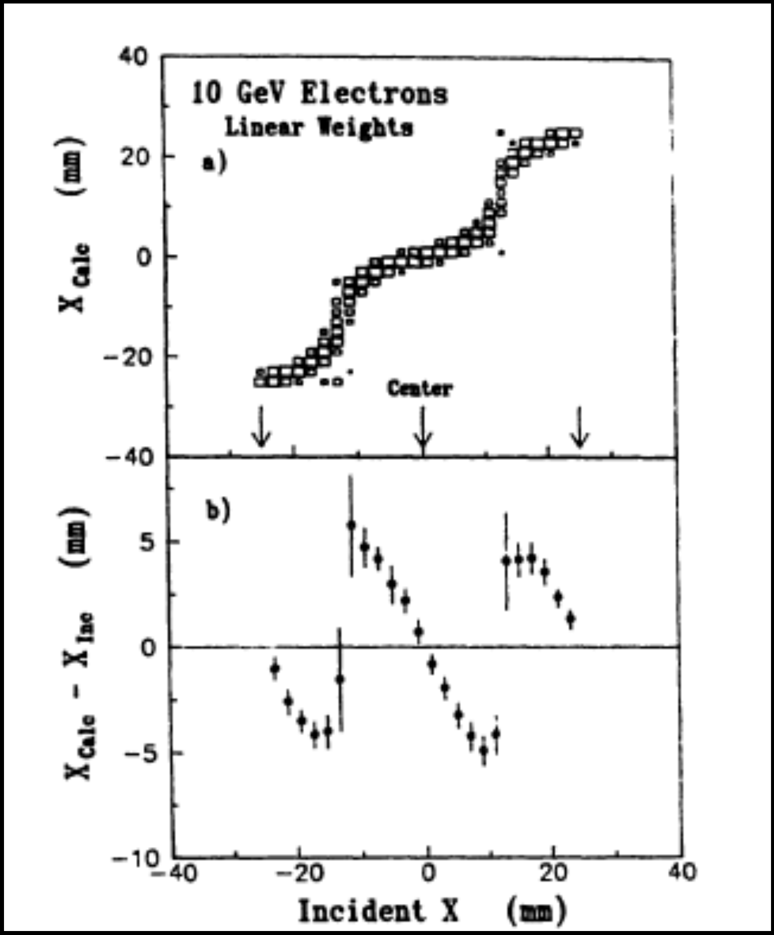
$$= <\eta>^2 - 2<\eta>^2 + <\eta^2> = <\eta^2> - <\eta>^2$$

SO THAT SHOULD LOOK FAMILIAR…

- We can then see that this really is the variance about the mean in η. With appropriate weighting. You would normally weight this sort of calculation by the uncertainty in each value, but that's not what we do here.
- Which brings us to the second equation.
  - Here's an actual paper reference to where this comes from: Awes et al., NIM A311, p130-138.
  - Also more directly for CMS, CMS Note 2001/034.

FROM NIM:

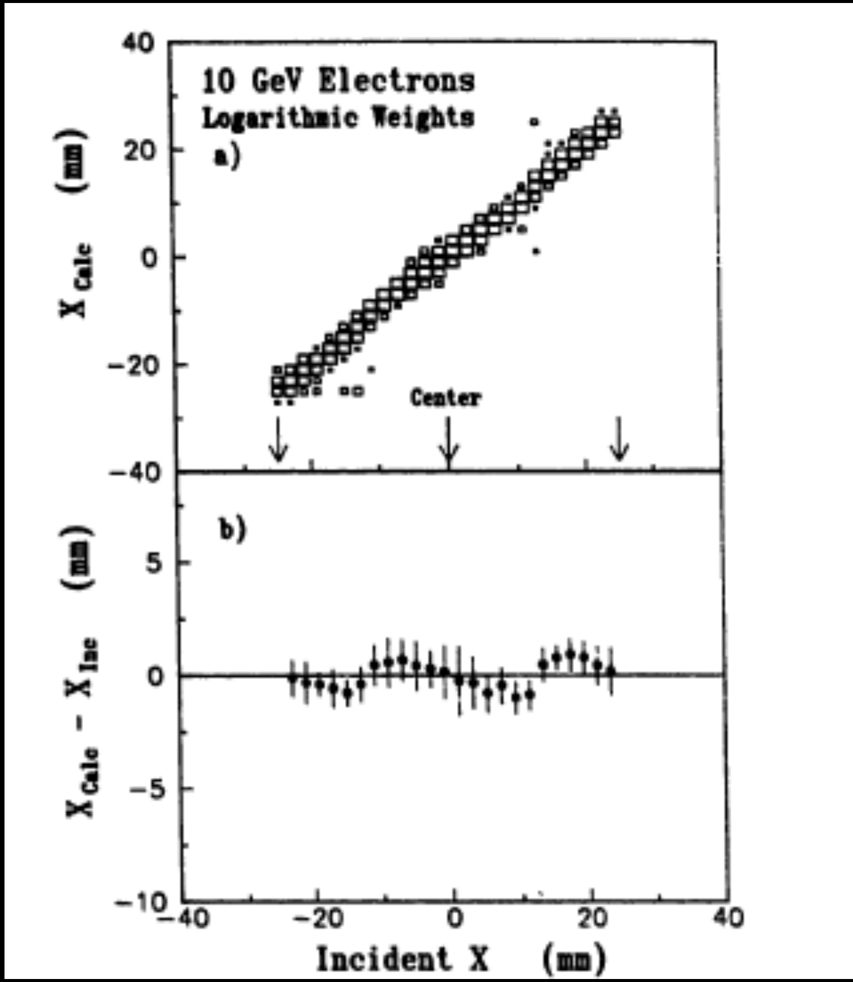
- This was originally written for a different crystal calorimeter, the one used in the L3 experiment at LEP. This plot is from their simulation which shows what happens if you calculate the position just using linear weights with energy.
  - This estimation counts the central crystal too heavily, and results in a “drawing” of the position.
  - This is a known problem.



SINCE YOU EXPECT…

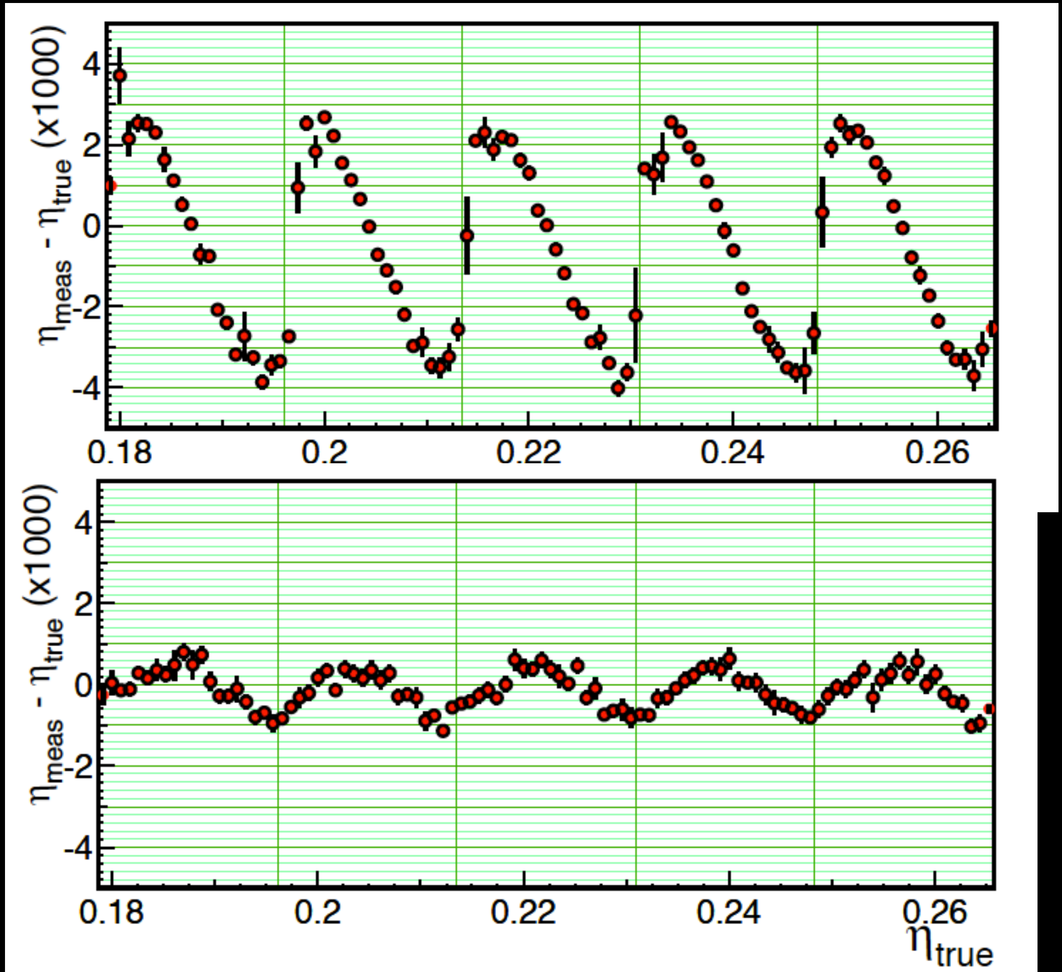
- One expects a gaussian falloff of the electromagnetic shower, which suggests a logarithmic weighting. You can see the result on the side.
- Their equation:

$$w_i = \max\left\{0, \left[W_0 + \ln\left(\frac{E_i}{E_T}\right)\right]\right\}$$



WHAT'S W_0?

- The short answer is that it has to be optimized. Here are two plots from the CMS note I referenced: Our current default is 4.7



Linear

Log-weighted, w_0=4.2

IN REALITY

- As long as the mean η was calculated with the same weighting, all of this logically hangs together.
  - The "I" in $\sigma_{i\eta i\eta}$ stands for "integer", the calculations are carried out in the twenty-five crystals, using their relative crystal indices.
  - This centers the matrix on the highest energy crystal, and then proceeds to ignore the rest of the results from the clustering.
- For some reason I've never figured out, the weighting for the position is NOT calculated consistently. Maybe someday.
  - This is weird for multiple reasons, especially considering that the actual Photon position uses the log-weighting.
  - For now, the mean η in this calculation is just straight energy weighted. The distribution actually does change because of this.

IN ACTION:

- Actually calculating the shower width by hand isn't as difficult as it sounds.
  - Don't get me wrong, it's not trivial. First you have to calculate the energy weighted position, and then you need to calculate weights for each of the crystals.
  - It should go without saying, you ignore negative energy crystals here.
  - Here are a couple of simple examples as to how this goes…

SIMPLE CASE: ONE CRYSTAL HAS 85% OF ENERGY THE OTHER HAS 15%.

$$\bar{\eta} = (0*0.85 + 1*0.15) = 0.15$$

$$w_0 = 4.7 + \ln(0.85) = 4.54; w_1 = 4.7 + \ln(0.15) = 2.80$$

$$\sigma^2_{i\eta i\eta} = \frac{(0-0.15)^2 * 4.54 + (1-0.15)^2 * 2.80}{4.54 + 2.80} = 0.289$$

$$\sigma^2_{i\eta i\eta} = 0.289 * (0.0174)^2$$ Convert to eta space

$$\sigma_{i\eta i\eta} = 0.00936$$

SIMPLE CASE: ONE CRYSTAL HAS 51% OF ENERGY THE OTHER HAS 49%.

$$\bar{\eta} = (0 * 0.51 + 1 * 0.49) = 0.49$$

$$w_0 = 4.7 + \ln(0.51) = 4.02; w_1 = 4.7 + \ln(0.49) = 3.99$$

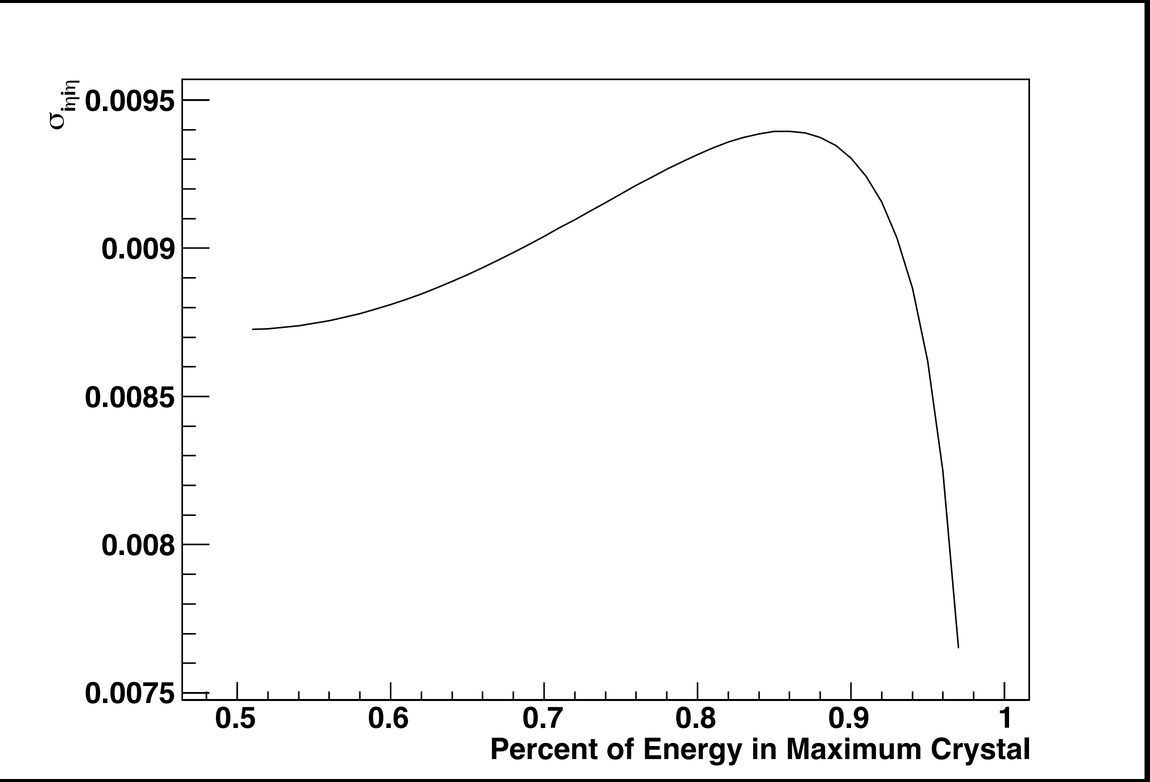
$$\sigma^2_{i\eta i\eta} = \frac{(0 - 0.49)^2 * 4.02 + (1 - 0.49)^2 * 3.99}{4.02 + 3.99} = 0.250$$

$$\sigma^2_{i\eta i\eta} = 0.250 * (0.0174)^2$$ Convert to eta space

$$\sigma_{i\eta i\eta} = 0.0087$$

AS AN EXAMPLE:

- Take a somewhat trivial model, say: all the energy from the photon or electron is shared only between only two crystals.
- You can then calculate what happens to $\sigma_{i\eta i\eta}$ as one varies the fraction that each crystal receives.
- Not as simple as one might think. There's no conservation of the weights used, so you could end up with some very different values, even with a simple model.



ENDCAP

- Just as a BTW—one notices that the shower shape is distinctly different in the endcap.
- This is effectively due to two reasons:
  - The conversion from crystal space to η space is different in the endcap (0.0447 in EE as opposed to 0.01745 in the EB)
  - The crystal coordinates aren't in iη/iϕ, they're in ix/iy, so an approximation is made.
- For this reason, you should really NEVER put EB and EE values of this variable in the same plot. It isn't the same thing, even if the calculation in all other ways is identical.

IN ACTION

- The best way to understand how this calculation works is to actually DO it.
- So that's what we're going to do. Take the first Hybrid clustering exercise, and use that 5x5.
- After you've calculated that value, you can open the interactive spreadsheet, and play around with the energy distribution, dynamically calculating the width.