

2.2. Construction of the transition matrix

In order to use this method for a given distribution π , we must construct a Markov chain \mathbf{P} with π as its stationary distribution. We now describe a general procedure for doing this which contains as special cases the methods which have been used for problems in statistical mechanics, in those cases where the matrix \mathbf{P} was made to satisfy the reversibility condition that for all i and j

$$\pi_i p_{ij} = \pi_j p_{ji}. \quad (4)$$

The property ensures that $\sum \pi_i p_{ij} = \pi_j$, for all j , and hence that π is a stationary distribution of \mathbf{P} . The irreducibility of \mathbf{P} must be checked in each specific application. It is only necessary to check that there is a positive probability of going from state i to state j in some finite number of transitions, for all pairs of states i and j .

We assume that p_{ij} has the form

$$p_{ij} = q_{ij} \alpha_{ij} \quad (i \neq j), \quad (5)$$

with

$$p_{ii} = 1 - \sum_{j \neq i} p_{ij},$$

where $\mathbf{Q} = \{q_{ij}\}$ is the transition matrix of an arbitrary Markov chain on the states $0, 1, \dots, S$ and α_{ij} is given by

$$\alpha_{ij} = \frac{s_{ij}}{1 + \frac{\pi_i q_{ij}}{\pi_j q_{ji}}}, \quad (6)$$