

where  $s_{ij}$  is a symmetric function of  $i$  and  $j$  chosen so that  $0 \leq \alpha_{ij} \leq 1$  for all  $i$  and  $j$ . With this form for  $p_{ij}$  it is readily verified that  $\pi_i p_{ij} = \pi_j p_{ji}$ , as required. In order to simulate this process we carry out the following steps for each time  $t$ :

(i) assume that  $X(t) = i$  and select a state  $j$  using the distribution given by the  $i$ th row of  $\mathbf{Q}$ ;

(ii) take  $X(t+1) = j$  with probability  $\alpha_{ij}$  and  $X(t+1) = i$  with probability  $1 - \alpha_{ij}$ .

For the choices of  $s_{ij}$  we will consider, only the quantity  $(\pi_j q_{ji})/(\pi_i q_{ij})$  enters into the simulation and we will henceforth refer to it as the test ratio.