Speed of light $c = 1$ invariant:

Inertial Frames Σ, Σ'

$$x^1 = x^0, \qquad x'^1 = x'^0$$

$$x'^1 = 0 \quad \text{at} \quad x^1 = \underset{\uparrow\ \text{velocity}}{\beta}\, x^0 = \frac{v}{c}\, x^0$$

Transformations which leave

$$(x^1)^2 - (x^0)^2 \quad \text{invariant:}$$

$$\begin{pmatrix} i x'^0 \\ x'^1 \end{pmatrix} = \begin{pmatrix} \mathrm{ch}(\xi) & -i\,\mathrm{sh}(\xi) \\ i\,\mathrm{sh}(\xi) & \mathrm{ch}(\xi) \end{pmatrix} \begin{pmatrix} i x^0 \\ x^1 \end{pmatrix}$$

$$i x'^0 = i x^0\, \mathrm{ch}\xi - i x^1\, \mathrm{sh}\xi$$

$$x'^1 = -x^0\, \mathrm{sh}\xi + x^1\, \mathrm{ch}\xi$$

Meaning of ξ from $x'^1 = 0$:

$$x^1\, \mathrm{ch}\xi = x^0\, \mathrm{sh}\xi$$

$$x^1 = x^0\, \mathrm{th}\xi = \beta x^0$$

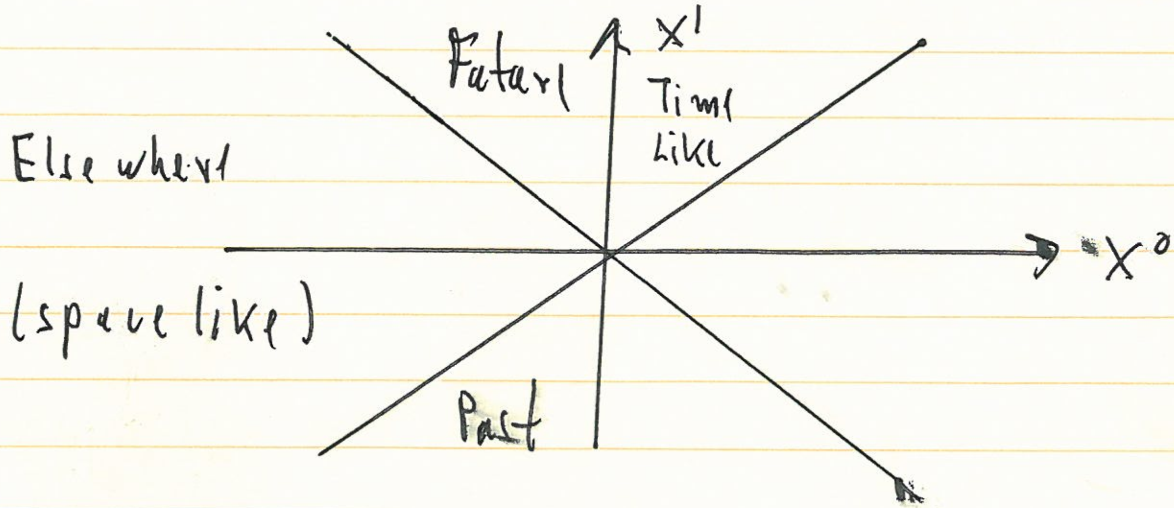
$\beta \underset{<1}{=} \mathrm{th}\xi$, ξ called "rapidity".

$$(x^{\prime 1})^2 + (ix^{\prime 0})^2 = (x^{\prime 1})^2 - (x^{\prime 0})^2 \quad (2)$$

$$= -(x^0)^2 ch^2\xi + \cancel{2x^0x^1 ch\xi sh\xi} - (x^1)^2 sh^2\xi$$

$$+ (x^0)^2 sh^2\xi - \cancel{2x^0x^1 sh\xi ch\xi} + (x^1)^2 ch^2\xi$$

$$= (x^1)^2 - (x^0)^2 = (x^1)^2 + (ix^0)^2$$



Minkowski Space

Let $x^0 < y^0$ and

$$iy^{\prime 0} = iy^0 ch\xi - iy^1 sh\xi$$

$$y^{\prime 1} = -y^0 sh\xi + y^1 ch\xi$$

Transformation to Σ', so that $\underline{\underline{x^{\prime 1} = y^{\prime 1}}}$:

$$-x^0 sh\xi + x^1 ch\xi = -y^0 sh\xi + y^1 ch\xi$$

$$(y^0 - x^0) sh\xi = (y^1 - x^1) ch\xi$$

$$\left|\frac{(y^1 - x^1)}{(y^0 - x^0)}\right| = |th\,\xi| < 1 \quad \text{Time like}$$

(within forward or backward cone).

Transformation to Σ', so that $\underline{\underline{x'^0 = y'^0}}$:

$$x^0 \, ch\,\xi - x^1 sh\,\xi = y^0 ch\,\xi - y^1 sh\,\xi$$

$$(y^0 - x^0)\, ch\,\xi = (y^1 - x^1)\, sh\,\xi$$

$$\left|\frac{(y^0 - x^0)}{(y^1 - x^1)}\right| = |th\,\xi| < 1 \quad \underline{\underline{\text{Space-Like.}}}$$