Mathematical Physics — PHZ 3113 Final (May 2, 2013)

Einstein's convention is used in the following problems and ϵ_{ijk} is the Levi-Civita tensor.

1. Calculate

$$\epsilon_{12k} \epsilon_{21k} =$$

$$\epsilon_{ij3} \epsilon_{ij3} =$$

$$\epsilon_{ijk} \epsilon_{ijk} =$$

2. Rewrite the expression

 $\epsilon_{ijk} \epsilon_{klm} \hat{x}_i \partial_j (A_l B_m),$ where the \hat{x}_i are Cartesian unit vectors and $\partial_j = \frac{\partial}{\partial x_j}$, into

- (A) A vector product.
- (B) A sum of scalar products.
- 3. Consider spherical coordinates. Calculate the time derivative

$$\dot{\hat{ heta}} = \frac{d\,\hat{ heta}}{d\,t}$$

of the unit vector $\hat{\theta}$ and express the result in spherical coordinates.

4. Find the eigenvalues and (normalized) eigenvectors of the matrix

$$\begin{pmatrix} +1 & -1 & 0 \\ -1 & +2 & -1 \\ 0 & -1 & +1 \end{pmatrix}$$