

Mathematical Physics — PHZ 3113
Divergence, (January 25, 2013)

1. Solution

$$\nabla \cdot \vec{r} = \sum_{i=1}^3 \frac{\partial x_i}{\partial x_i} = 3. \quad (1)$$

2. Solution

$$\begin{aligned} \nabla \cdot \vec{r} f(r) &= f(r) \nabla \cdot \vec{r} + \vec{r} \cdot \nabla f \\ &= 3 f(r) + \vec{r} \cdot \hat{r} \frac{df}{r} = 3 f(r) + r \frac{df}{r}. \end{aligned} \quad (2)$$

3. Example: Calculate (2) for $f(r) = r^{n-1}$.

Solution:

$$\begin{aligned} \nabla \cdot \vec{r} r^{n-1} &= 3 r^{n-1} + r \frac{dr^{n-1}}{dr} \\ &= 3 r^{n-1} + (n-1) r^{n-1} = (n+2) r^{n-1}. \end{aligned} \quad (3)$$

4. Solution for $\vec{r} \neq \vec{0}$

$$\nabla \cdot \vec{E} = (-2 + 2) \frac{q \hat{r}}{4\pi \epsilon_0 r^2} = 0. \quad (4)$$