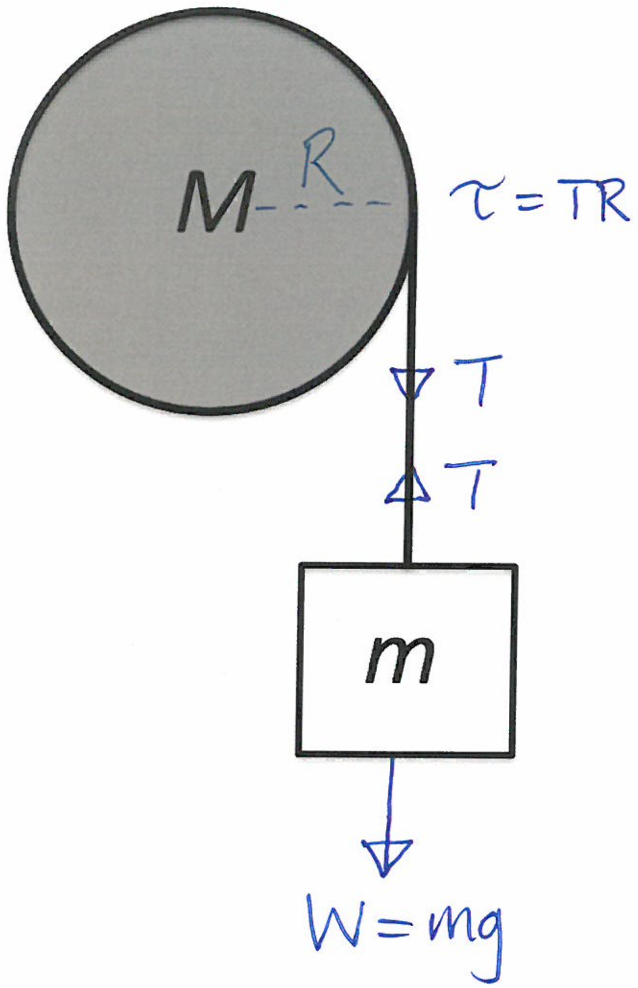
Rotational Newton's Law problem



2nd law on mass

$$W - T = ma$$

$$\Rightarrow T = W - ma$$

$$= m(g-a)$$

On disk

$$\tau = I\alpha = I\frac{a}{R}$$

$$\downarrow$$

$$TR = I\frac{a}{R}$$

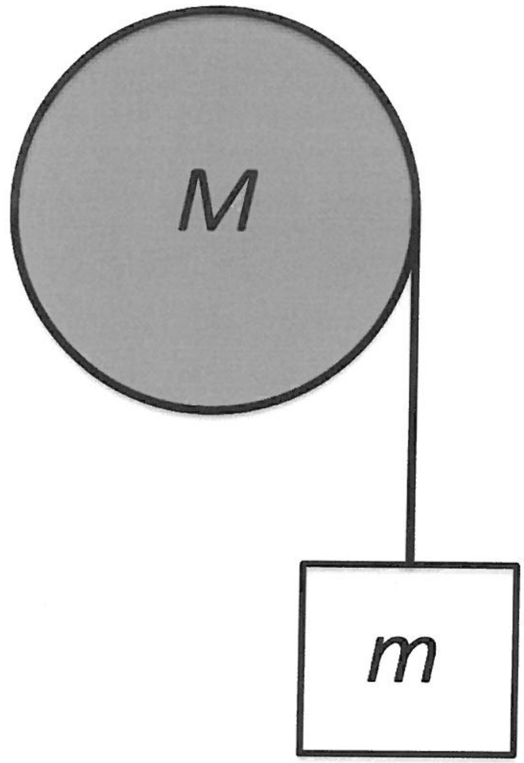
$$\Rightarrow T = \frac{I}{R^2}a$$

Solving for a: $\frac{I}{R^2}a = mg - ma$

$$\Rightarrow \left(1 + I/mR^2\right)a = g$$

So $a = \dfrac{g}{1 + I/mR^2}$

Since $I_{disk} = \frac{1}{2}MR^2$ $= \dfrac{g}{1 + \frac{m}{2M}}$



M
m

$$I_{scythe} = I_{blade} + I_{handle}$$

$$I_{blade} \cong ML^2$$

parallel axis theorem

$$I_{handle} = I_{cm} + Md^2 \qquad d = \frac{L}{2}$$

$$= \frac{1}{12}ML^2 + M\frac{L^2}{4} = \frac{1}{3}ML^2$$

$$I_{scythe} = ML^2 + \frac{1}{3}ML^2$$

$$= \frac{4}{3}ML^2$$

Rotational/Rolling motion

Hoop: $f = 1$

Disk: $f = 1/2$

Solid Sphere: $f = 2/5$

Hollow Sphere: $f = 2/3$

Then $K = (1+f) \times \frac{1}{2} m v^2$

Then $U \rightarrow K$ etc...

just re-defining K.

Otherwise problems exactly the same!

Three round objects, (i) a disk, (ii) a hoop, and (iii) a solid sphere, are released from the top of the ramp shown below. The objects have the same and mass, equal to 5 kg. The release point is 1 m above the horizontal section of the track. Calculate the resultant velocities of the three objects when they reach the bottom. You should assume that they roll without slipping.

Key Idea:

$$K = \frac{1}{2}Mv^2 \times (1+f)$$

Cons. Mech. Eng. $\Delta U + \Delta K = 0$

$$\Rightarrow \quad \Delta K = -\Delta U$$
$$= +mgh$$

$$\Rightarrow \quad \frac{1}{2}\cancel{M}v^2(1+f) = \cancel{m}gh$$

$$v = \sqrt{\frac{2gh}{1+f}}$$

Note: usual result if $f=0$, i.e., no rolling

$f_{hoop} = 1$; $f_{disk} = \frac{1}{2}$; $f_{sphere} = \frac{2}{5}$

$v_{hoop} = \sqrt{\frac{2gh}{2}} = \sqrt{gh}$; $v_{disk} = \sqrt{\frac{2gh}{3/2}} = \sqrt{\frac{4gh}{3}}$

$v_{sphere} = \sqrt{\frac{2gh}{7/5}} = \sqrt{\frac{10gh}{7}}$ Ratio 1:1.15:1.20