2D Motion - 1

Simple projectile motion (1).

Maximum height:

$$v_x^0 = v^0 \cos(\theta)\,, \quad v_y^0 = v^0\, cos(\theta) \quad v_y = v_y^0 - g\, t$$

and $v_y = 0$ for $y = y_{\rm max} \;\Rightarrow\; t_{\rm max} = v_y^0/g$. Therefore,

$$y_{max} = v_y^0\, t_{\rm max} - g\, t_{\rm max}^2/2\,.$$

Time at which the maximum height is reached: Already calculated.

Off the cliff (2): Insert t from the second equality into the first:

$$x = v_0\, t \;\; \text{and} \;\; H = g\, t^2/2\,.$$

2D Motion - 2

Horizontal Distance (3).

First, use

$$g\, t_{\text{max}} - v^0 = 0 \quad \text{and} \quad H = v^0\, t_{max} - \frac{1}{2}\, g\, (t_{\text{max}})^2$$

to express v_0 as function of g and H. Components:

$$v_x^0 = v^0 \cos(\phi)\,, \quad v_y^0 = v^0 \sin(\phi)\,.$$

From $g\, t_{\text{max}}^{\phi} - v_y^0 = 0$ the horizontal distance follows:

$$d = 2\, t_{\text{max}}^{\phi}\, v_x^0 = \frac{2}{g}\, (v_0)^2 \cos(\phi)\, \sin(\phi)\,.$$

Determine ϕ_{max} from the derivative of d,

$$\left.\frac{d\, d}{d\, \phi}\right|_{\phi=\phi_{\text{max}}} = 0\,,$$

and eliminate v_0 in terms of g and H.

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Horizontal Range (4).

$$v_y^0 = \sqrt{2\,h\,g}\,, \quad t = \frac{v_y^0}{g}\,, \quad R = 2\,t\,v_x^0\,, \quad v^0 = \sqrt{(v_y^0)^2 + (v_x^0)^2}\,.$$

Catapult (5).

With g, H, D, and t given:

$$v_x^0 = \frac{D}{t} = \frac{x_f}{t}\,, \quad \text{and} \quad 0 = H + v_y^0\,t - \frac{1}{2}\,g\,t^2 \quad \Rightarrow \quad v_y^0\,.$$

$\Rightarrow$ angle and speed. Heighest point:

$$0 = v_y^0 - g\,t_{max} \quad \Rightarrow \quad y_{\text{max}} = H + \cdots\,.$$

2D Motion - 5

Circular acceleration (6):

$$T = \frac{2\pi\, r}{v}\,, \quad \omega = \frac{2\pi}{T} = \frac{2\pi\, v}{r} = \frac{v}{r}\,.$$

$$x = +r\, \cos(\omega\, t)\,, \quad y = +r\, \sin(\omega\, t)\,,$$

$$v_x = -\omega\, r\, \sin(\omega\, t)\,, \quad v_y = +\omega\, r\, \cos(\omega\, t)\,,$$

$$a_x = -\omega^2\, r\, \cos(\omega\, t)\,, \quad a_y = -\omega^2\, r\, \sin(\omega\, t)\,,$$

Therefore,

$$\vec{r}^{\,2} = r^2\,, \quad \vec{v}^{\,2} = \omega^2\, r^2\,, \quad \vec{a}^{\,2} = \omega^4\, r^2 \quad \Rightarrow \quad a = \omega^2\, r = \frac{v^2}{r}\,.$$