

Momentum Conservation - 1

Mass Mass-Spring Collision (1):

$$m_1 v_1 = M v, \quad K_i = m_1 \frac{v_1^2}{2}, \quad K_f = M \frac{v^2}{2}, \quad \frac{K}{2} (\Delta x)^2 = K_f.$$

Freight Car-Carboose Collision (2). Equate the energies and use momentum conservation:

$$(1 - p) m_1 \frac{v_1^2}{2} = (m_1 + m_2) \frac{v^2}{2}, \quad v = \frac{m_1 v_1}{m_1 + m_2}.$$

Divide common factors out.

Ball-Bat Collision (3). As $m_{\text{bat}} \gg M = m_{\text{ball}}$, the rest frame of the bat can be taken as CM frame in which the elastic collision takes place, and the bat acts like a brick wall:

$$v'_i = -v_i, \quad M v'_i = -M v_i.$$

Then, transfer v'_f to v_f , the final velocity of the ball in the original frame. Part 2: v_f^2/v_i^2 .

Momentum Conservation - 2

Collision Between Two Balls (4). Steel balls:

$$h = r - r \cos(\theta).$$

Putty balls ($v_f = ? v_i$ from momentum conservation):

$$K_i = m \frac{v_i^2}{2} = M h g, \quad K_f = m \frac{v_f^2}{2} = 2 M H g \quad \Rightarrow \quad \frac{H}{h}.$$

Glancing Collision (5):

$$\begin{aligned} p_y^i &= 0 = \sin(\theta_1) m_1 v_1^f + \sin(\theta_2) m_2 v_2^f, \\ p_x^i &= m_1 v^i = \cos(\theta_1) m_1 v_1^f + \cos(\theta_2) m_2 v_2^f, \end{aligned} \quad (1)$$

with $\theta_2 < 0$, $m_1 = m$ and $m_2 = 2m$. Eliminate v_1^f in favor of v_2^f and solve for v_2^f with v^i given.

Momentum Conservation - 3

Elastic Head-On Collision (6):

$$m_1 v_1^i + m_2 v_2^i = m_1 v_1^f + m_2 v_2^f,$$

$$\frac{1}{2} m_1 (v_1^i)^2 + \frac{1}{2} m_2 (v_2^i)^2 = \frac{1}{2} m_1 (v_1^f)^2 + \frac{1}{2} m_2 (v_2^f)^2.$$

Example (you may have different numbers): $m_1 = m$, $v_1^i = 4v$, $m_2 = 2m$ and $v_2^i = -v$. Use the first equation to eliminate $(v_2^f)^2$ in the second. Solve the resulting quadratic equation for v_1^f . Which of the solutions to choose? Enter the speed which positive by definition.

Two Unequal Mass Balls (7): Meant is the situation where the heavy ball bounces back with velocity $-v$, while the lighter ball is still heading towards it with velocity $+v > 0$. Thus, the collision is reduced to the approach of the previous problem. Once v_1^f is found use

$$\frac{h'}{h} = \frac{(v_1^f)^2}{v^2}.$$