Angular Momentum - 1

Angular Momentum and Torque (1):

$$\vec{L} = \vec{r} \times M\,\vec{v}\,, \quad \vec{\tau} = \vec{r} \times \vec{F}\,.$$

Rotating Chair (2). Angular momentum conservation:

$$L = I_i\,\omega_i = I_f\,\omega_f\,.$$

Kinetic energy is not conserved:

$$\triangle K = \frac{1}{2}\,I_f\,\omega_f^2 - \frac{1}{2}\,I_i\,\omega_i^2\,.$$

Rotating Nucleus (3). Angular velocity:

$$L = I\,\omega \quad \text{with} \quad I = \frac{2}{5}\,M\,r^2\,.$$

The other questions:

$$v = \omega\,r\,, \quad K_{\text{rot}} = \frac{1}{2}\,I\,\omega^2\,, \quad \text{Ratio} = \frac{K_{\text{rot}}}{M\,c^2}\,.$$

Angular Momentum - 2

Children on a Merry-Go-Round (4). Rotational inertia:

$$I = 2\,M_c\,R^2 + \frac{M_b\,L^2}{12}$$

with M_c the mass of one child, M_b the mass of the board and $R = L/2$. Angular momentum: $L = I\,\omega$. Angular momentum conservation when the children are moving:

$$L = L' = I'\,\omega' \quad \text{with} \quad I' = 2\,M_c\left(\frac{R}{2}\right)^2 + \frac{M_b\,L^2}{12}\,.$$

As in (2) change in kinetic energy:

$$\triangle K = \frac{1}{2}\,I'\,\omega'^2 - \frac{1}{2}\,I\,\omega^2\,.$$

Putty on Turntable (5): 1. Angular momentum conservation:

$$I_0\omega_0 = I_f\omega_f \quad \text{with} \quad I_f = I_0 + m\,R^2\,.$$

2. Angular momentum carried away.

Angular Momentum - 3

Mass on Turntable (6). Energy and angular momentum are conserved:

$$\frac{1}{2}\,k\,x^2 = \frac{1}{2}\,m\,v^2 + \frac{1}{2}\,I\,\omega^2\,, \quad b\,m\,v = I\,\omega\,.$$

1. Insert $\omega = b\,m\,v/I$ in the first equation and solve for v. 2. Insert $v = I\,\omega/(b\,m)$ in the first equation and solve for ω.

Time-Dependent Torque (7):

$$L(t) = \int_0^t dt'\,\tau(t')\,.$$