Special and General Relativity (PHZ 4601/5606) Fall 2018 Solutions

Set 4

8. Euclidean and hyperbolic rotations.

Transformation of the 2D Euclidean rotation.

$$\begin{pmatrix} x'^{\,1} \\ i\,x'^{\,0} \end{pmatrix} = \begin{pmatrix} \cosh(\zeta) & i\,\sinh(\zeta) \\ -i\,\sinh(\zeta) & \cosh(\zeta) \end{pmatrix} \begin{pmatrix} x^1 \\ i\,x^0 \end{pmatrix}$$

and in components

$$\begin{aligned} x'^{\,1} &= \cosh(\zeta)\,x^1 - \sinh(\zeta)\,x^0\,, \\ i\,x'^{\,0} &= -i\,\sinh(\zeta)\,x^1 + i\,\cosh(\zeta)\,x^0\,, \end{aligned}$$

or, equivalently,

$$\begin{aligned} x'^{\,0} &= +\cosh(\zeta)\,x^0 - \sinh(\zeta)\,x^1\,, \\ x'^{\,1} &= -\sinh(\zeta)\,x^0 + \cosh(\zeta)\,x^1\,, \end{aligned}$$

$$\begin{pmatrix} x'^{\,0} \\ x'^{\,1} \end{pmatrix} = \begin{pmatrix} \cosh(\zeta) & -\sinh(\zeta) \\ -\sinh(\zeta) & \cosh(\zeta) \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \end{pmatrix}$$