Special and General Relativity (PHZ 4601/5606) Fall 2018 Solutions

Set 4

(9) Twin travel in 2D Minkowski space.

(a) $\beta^1_B = 4/5$.

(b) The proper time of B at B_0, the end of the first part of its travel, is $\sqrt{(5)^2 - (4)^2} = 3$.

(c) $2 \times 5 = 10$ is the time on the clock of A when A meets B again.

(d) $2 \times 3 = 6$ is the time on the clock of B when A meets B again.

(e) We have $\beta^2_A = -\beta^1_B = -4/5$ for the velocity of A in the inertial frame K_2.

(f) The coordinates of B_0 in K_2 are $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

(g) From the addition theorem of velocities we find

$$\beta^2_B = -\left(\frac{\beta^1_B + \beta^1_B}{1 + (\beta^1_B)^2} \right) = -\frac{8/5}{1 + 16/25} = -\frac{40}{41} \, .$$

(h) We may find the final meeting point by calculating where the straight lines on which A and B travel in K_2 meet or by performing the Lorentz transformation from K_1 to K_2. For simplicity of the equations we denote the coordinates in K_2 just by t and x without subscripts indicating K_2.

First method: $x = -4\,t/5$ for A and $x = -40\,(t-3)/41$ for B. Equating these equations gives

$$\begin{aligned} 4\,t/5 &= 40\,t/41 - 120/41 \\ 41 \times 4\,t &= 5 \times 40\,t - 5 \times 120 \\ 164\,t &= 200\,t - 600 \end{aligned}$$

with the result $t = 50/3$. Therefore, $x = -(4 \times 50)/(3 \times 5) = -40/3$. Together: The final space-time point in K_2 is

$$\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} 50/3 \\ -40/3 \end{pmatrix} .$$

Second method (Lorentz transformation):

$$\gamma = \sqrt{\frac{1}{1-\beta^2}} = \sqrt{\frac{1}{1-(4/5)^2}} = \sqrt{\frac{5^2}{5^2-4^2}} = \frac{5}{3} \quad \text{and} \quad \beta\gamma = \frac{4}{3}\,.$$

So, we find for the position of the final point in K_2

$$\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} 10 \\ 0 \end{pmatrix} = \begin{pmatrix} 5/3 & -4/3 \\ -4/3 & 5/3 \end{pmatrix} \begin{pmatrix} 10 \\ 0 \end{pmatrix} = \begin{pmatrix} 50/3 \\ -40/3 \end{pmatrix}\,.$$

(i) See the figure.

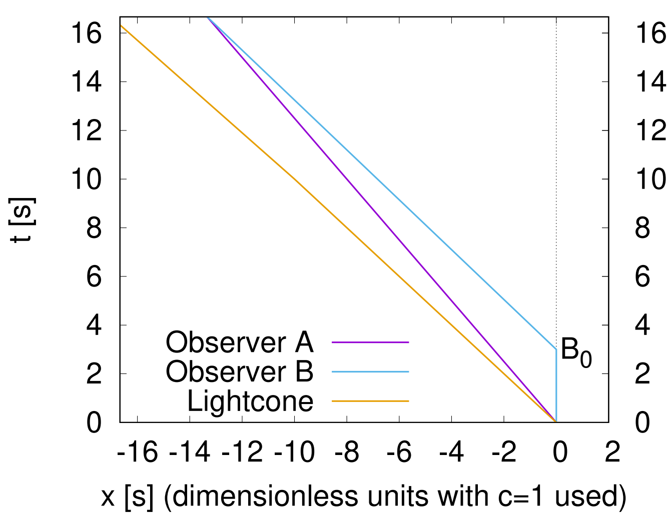

Figure: Travel of the inertial frame K_1 translated to the inertial frame K_2.