Special and General Relativity (PHZ 4601/5606) Fall 2018 Solutions

Set 11

26. Are $t = var$, $x^i = const$ lines geodesics?

The metric is $d\vec{s}^2 = dt^2 - dl^2$ and $dx^i = 0$, $t = var$ hold.
Hence, $ds^2 = dt^2 \Rightarrow s = t$. So, $\dot{t} = 1$, $\ddot{t} = 0$ and $\dot{x}^i = 0$, $\ddot{x}^i = 0$.
Now, let us check the geodesic equations of $L = \dot{t}^2 - g_{ij}\,\dot{x}^i\,\dot{x}^j$:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{t}} - \frac{\partial L}{\partial t} = 0 \quad \Rightarrow \quad 2\,\ddot{t} - g_{ij,4}\,\dot{x}^i\,\dot{x}^j = 0$$

which is solved by $\ddot{t} = 0$, $\dot{x}^i = 0$. Next,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}^i} - \frac{\partial L}{\partial x^i} = 0 \quad \Rightarrow \quad -2\,\frac{d}{dt}\left(g_{ij}\,\dot{x}^j\right) - g_{jk,i}\,\dot{x}^j\,\dot{x}^k = 0$$

because of $\dot{x}^i = 0$. So, the geodesic equations are satisfied.