Special and General Relativity (PHZ 4601/5606) Fall 2018 Solutions

Set 11

27. Some relations between covariant and ordinary differentiation

For vectors:

$$A_{i:j} = A_{i,j} + A_a \, \Gamma^a_{ij} \quad \text{and} \quad \Gamma^a_{ij} = \Gamma^a_{ji}$$

holds. Therefore,

$$A_{i:j} - A_{j:i} = A_{i,j} - A_{j,i} \, .$$

For antisymmetric tensors: Adding the equations

$$T_{ij:k} = T_{ij,k} - T_{aj} \, \Gamma^a_{ik} - T_{ia} \, \Gamma^a_{jk} \, ,$$
$$T_{jk:i} = T_{jk,i} - T_{ak} \, \Gamma^a_{ji} - T_{ja} \, \Gamma^a_{ik} \, ,$$

gives

$$T_{ij:k} + T_{jk:i} = T_{ij,k} + T_{jk,i} - T_{ia} \, \Gamma^a_{jk} - T_{ak} \, \Gamma^a_{ji} \, , \tag{1}$$

where two terms cancelled one another due to antisymmetry of T_{ij}. Adding now to (1)

$$T_{ki:j} = T_{ki,j} - T_{ai} \, \Gamma^a_{kj} - T_{ka} \, \Gamma^a_{ji} \, ,$$

using symmetry and antisymmery, gives the desired result

$$T_{ij:k} + T_{jk:i} + T_{ki:j} = T_{ij,k} + T_{jk,i} + T_{ki,j} \, .$$