Special and General Relativity (PHZ 4601/5606) Fall 2018 Solutions

Set 12

31. Towards the Schwarzschild metric.

(a) We have

$$a = a(r) = -e^{B(r)} = a(x^1)\,, \quad b = b(r) = -r^2 = b)(x^1)\,,$$
$$c = c(r,\theta) = -r^2\,\sin^2\theta = c(x^1, x^2)\,, \quad d = d(r) = e^{A(r)} = d(x^1)\,.$$

Therefore, $0 = a_2 = a_3 = a_4$ and the remaining part of R_{11} is

$$\begin{aligned} R_{11} \;&=\; +\,\beta\, b_{11} + \gamma\, c_{11} + \delta\, d_{11} \\ &\quad -\,\beta^2\, b_1^2 - \gamma^2\, c_1^2 - \delta^2\, d_1^2 \\ -\alpha\, a_1 &\;(\;+\,\beta\, b_1 + \gamma\, c_1 + \delta\, d_1\;\;) \end{aligned}$$

(b) For the derivatives we find

$$a_1 = -B_1\, e^B\,, \quad b_1 = -2r\,, \quad c_1 = -2r\,\sin^2\theta\,, \quad d_1 = A_1\, e^A\,,$$
$$b_{11} = -2\,, \quad c_{11} = -2\,\sin^2\theta\,, \quad d_{11} = A_{11}\, e^A + (A_1)^2\, e^A\,,$$

and R_{11} becomes

$$\begin{aligned} R_{11} = R_{rr} \;&=\; +\,\frac{1}{r^2} + \frac{1}{r^2} + \frac{A_{11}}{2} + \frac{(A_1)^2}{2} \\ &\quad -\,\frac{1}{r^2} - \frac{1}{r^2} - \frac{(A_1)^2}{4} \\ -\frac{B_1}{2} &\;(\;+\,\frac{1}{r} \;+\; \frac{1}{r} \;+\; \frac{A_1}{2}\;) \\ &=\; \frac{A_{11}}{2} - \frac{A_1\, B_1}{4} + \frac{(A_1)^2}{4} - \frac{B_1}{r}\,. \end{aligned}$$