

Special and General Relativity (PHZ 4601/5606) Fall 2018 Solutions

Set 9

22. Rindler 9.8: Transformation to a static metric.

Completion of the square gives

$$d\vec{s}^2 = x^2 (dt - 3x^2 y dx - x^3 dy)^2 - 9x^6 y^2 dx^2 - x^8 dy^2 - dx^2.$$

Now we substitute $t' = t + f(x, y)$ and want

$$dt' = dt + df = dt + f_x dx + f_y dy \quad \text{with} \quad f_x = -3x^2 y \quad \text{and} \quad f_y = -x^2.$$

Integrating f_x gives

$$\int f_x dx = - \int 3x^2 y dx = -x^3 y + g(y),$$

which yields $f_y = -x^3 + g'$ so that $g' = 0$ and $g(y)$ is a constant, and we are free to choose $g(y) = 0$. Hence,

$$dt' = dt - 3x^2 y dx - x^3 dy$$

holds and our final result is the static metric

$$d\vec{s}^2 = x^2 dt'^2 - 9x^6 y^2 dx^2 - x^8 dy^2 - dx^2.$$