

Newton gravitational potential:

$$V = V_N(r) = \frac{h^2}{r^2} - \frac{2m}{r} . \quad (1)$$

Schwarzschild gravitational potential:

$$V = V_{GR}(r) = \frac{h^2}{r^2} - \frac{2m}{r} - \frac{2m h^2}{r^3} = V_N(r) - \frac{2m h^2}{r^3} . \quad (2)$$

Using gravitational units $G = c = 1$ and as mass unit [sm], the central (solar) mass m :

$$V_N(r) = \frac{h^2}{r^2} - \frac{2}{r} \quad \text{and} \quad V_{GR}(r) = \frac{h^2}{r^2} - \frac{2}{r} - \frac{2h^2}{r^3} . \quad (3)$$

Here h is the specific angular momentum and the number 2 carries the dimension of a mass. In these units we have $h \approx 10,000$ for the orbit of the earth about the sun (not included in the following pictures).

Consider a point mass at the center.

Small h : Barrier against falling into the black hole disappears.

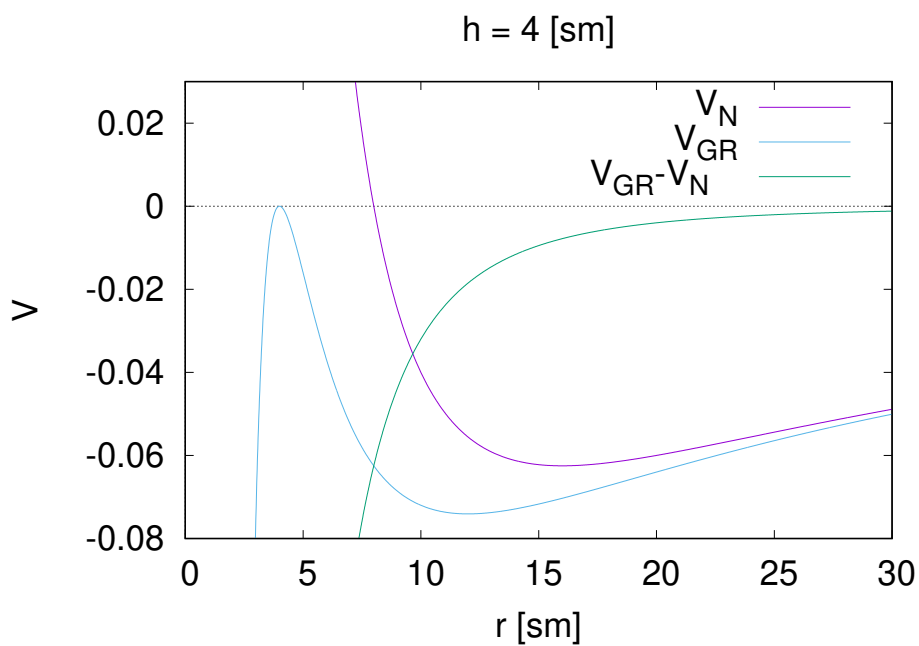
Large enough, increasing h : Barrier against falling into the black hole increases.

The maximum and minimum of the $V_{GR}(r)$ potential are at

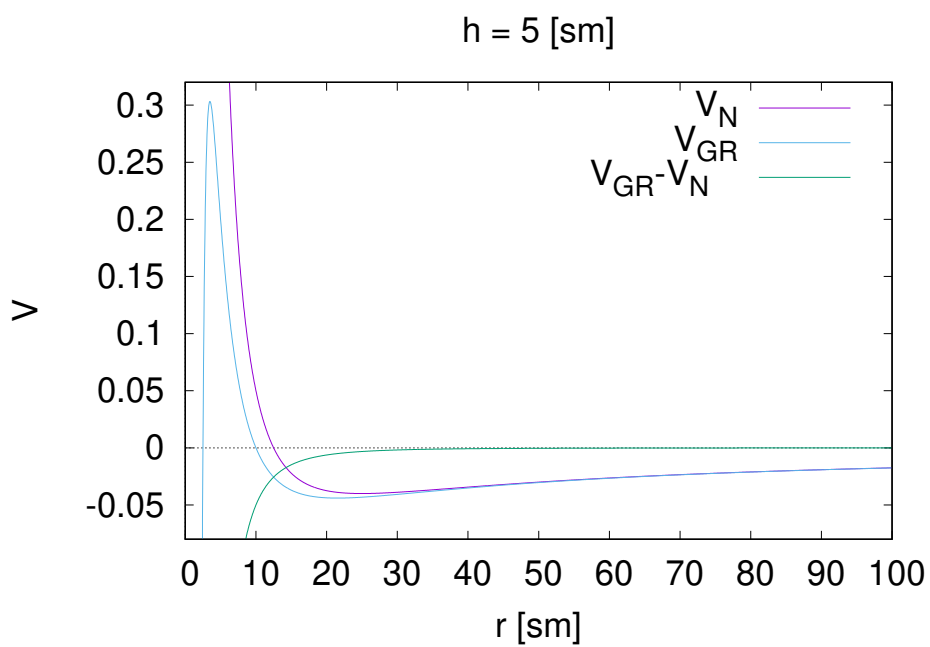
$$r_{\pm} = \frac{h^2}{2} \pm \sqrt{\left(\frac{h^2}{2}\right)^2 - 3h^2} , \quad (4)$$

where $V_{\min} = V_{GR}(r_+)$ and $V_{\max} = V_{GR}(r_-)$. From Eq. (4) it follows that there are no extrema for $h^2 \leq 12$.

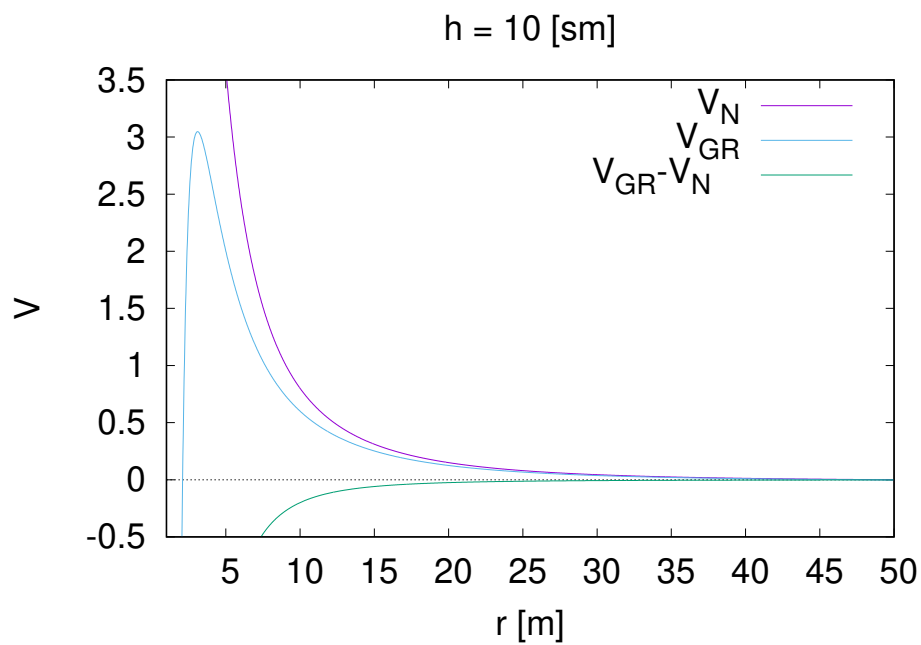
Increasing $h \geq 4$



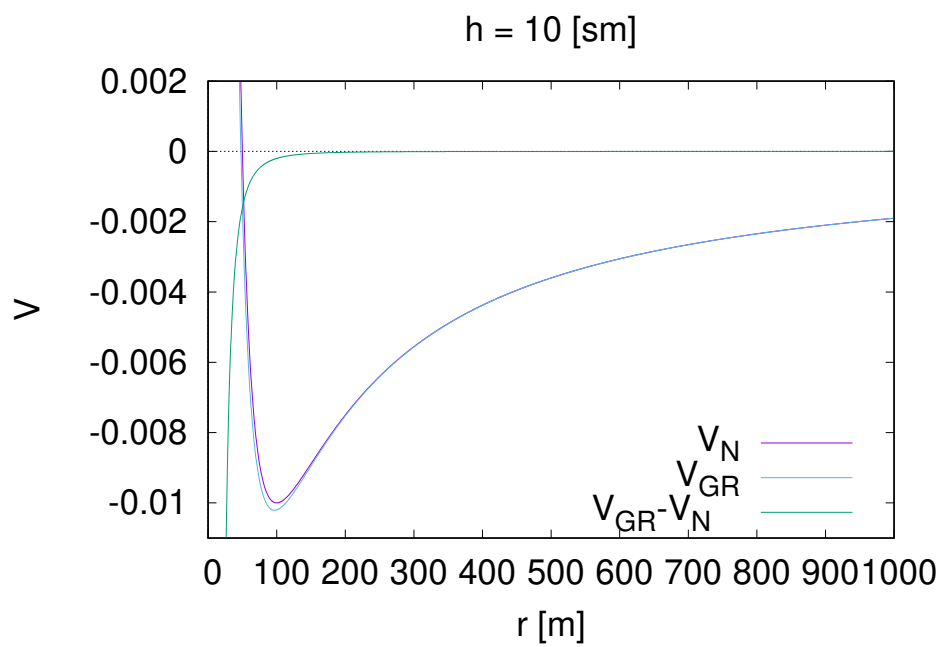
Gravitational Potential.



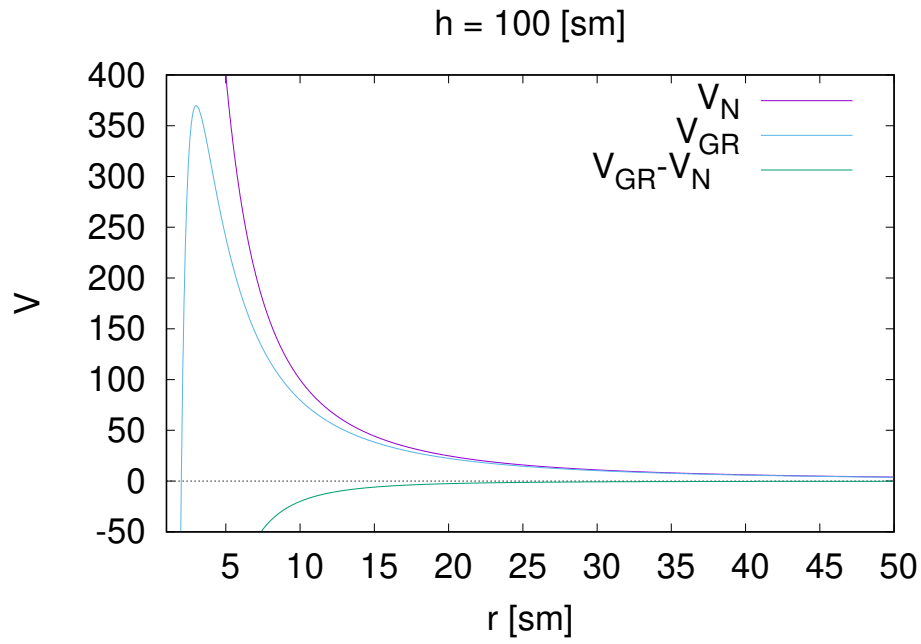
Gravitational Potential.



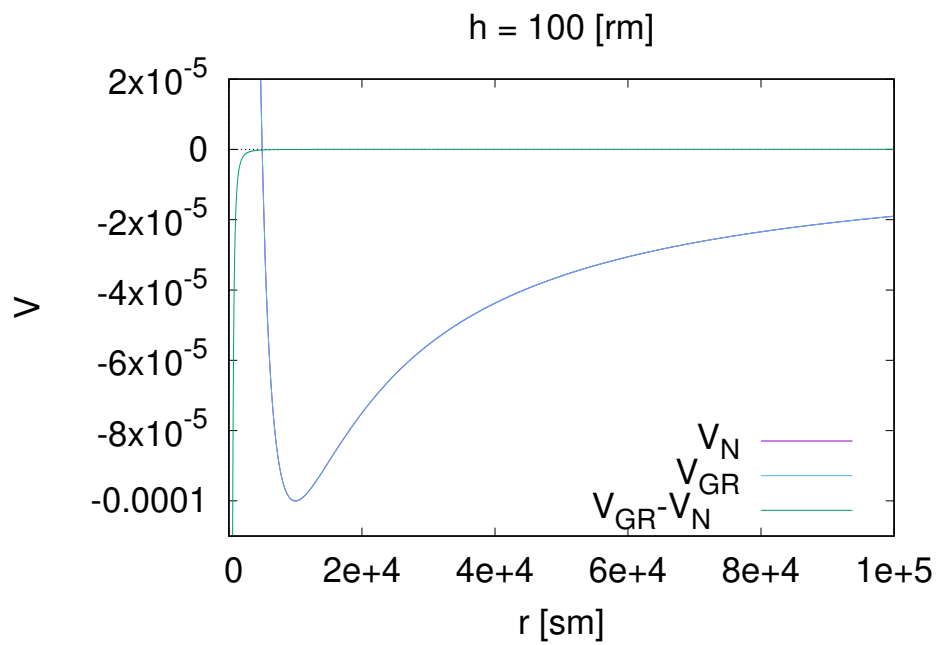
Schwarzschild Singularity.



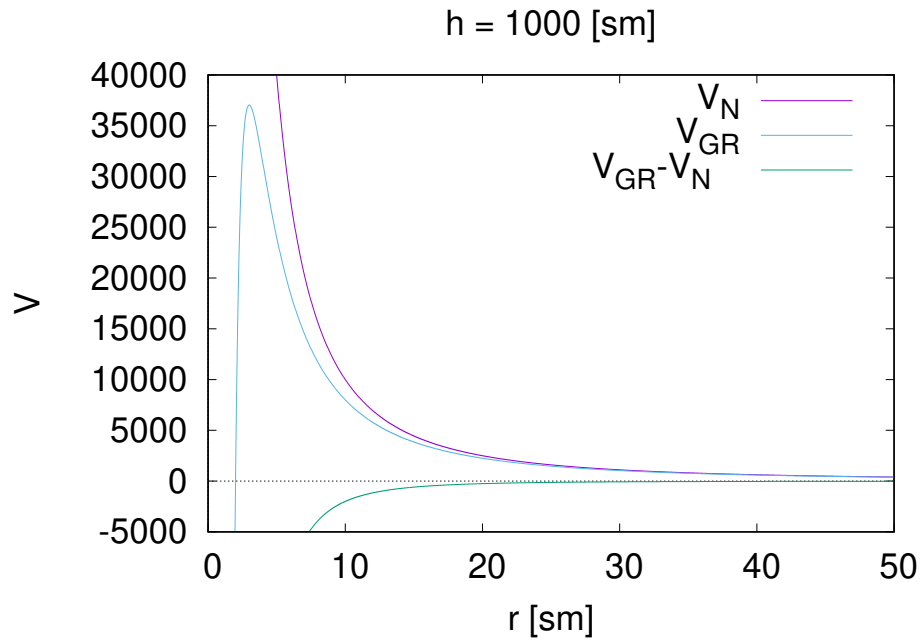
Newton Potential.



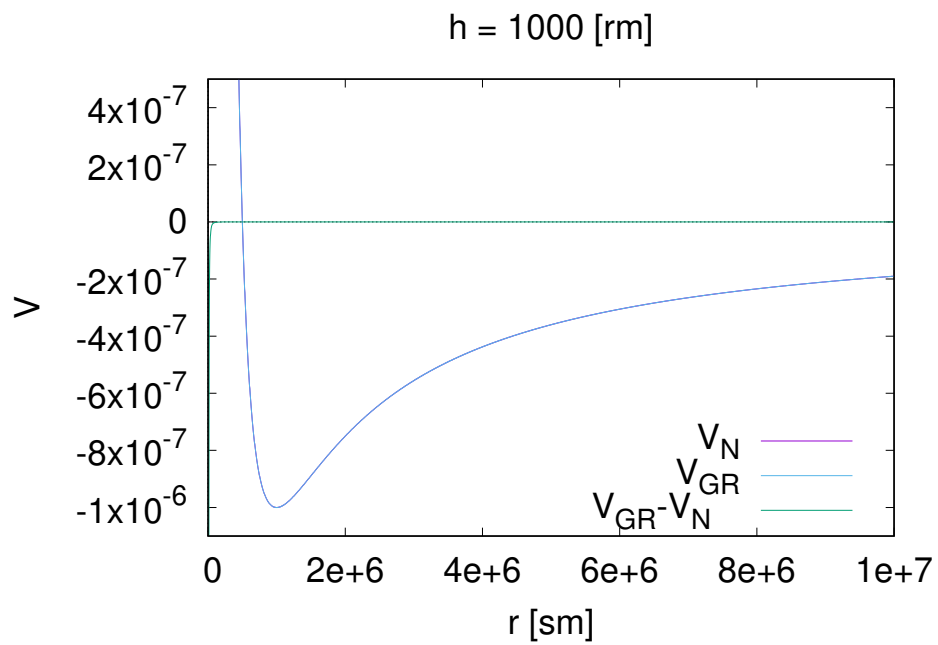
Schwarzschild Singularity.



Newton Potential.

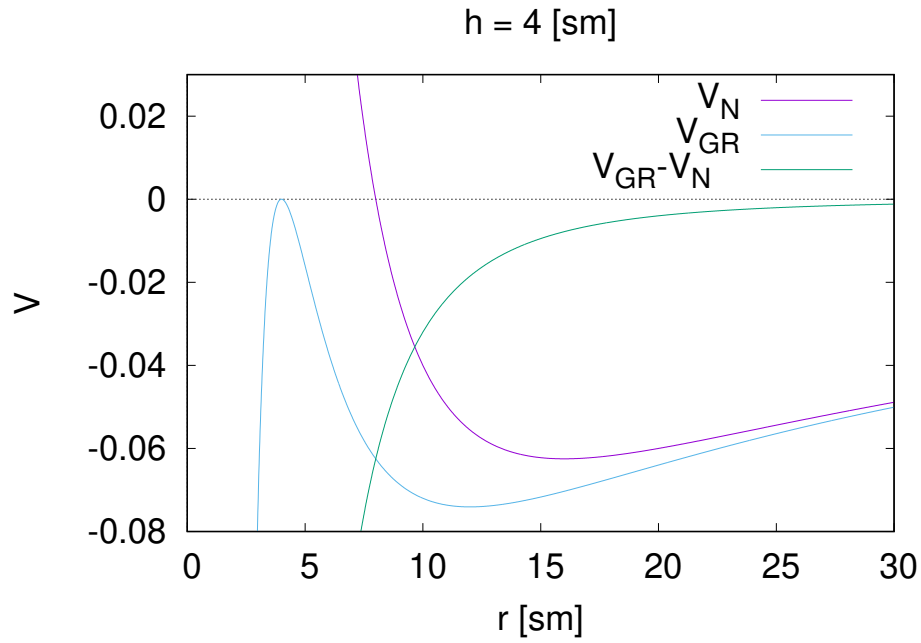


Schwarzschild Singularity.

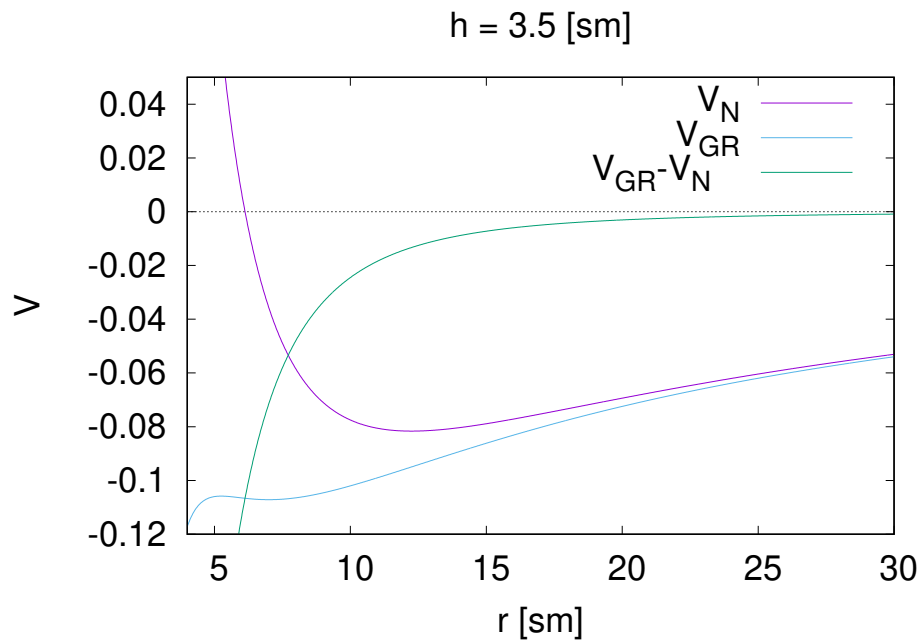


Newton Potential.

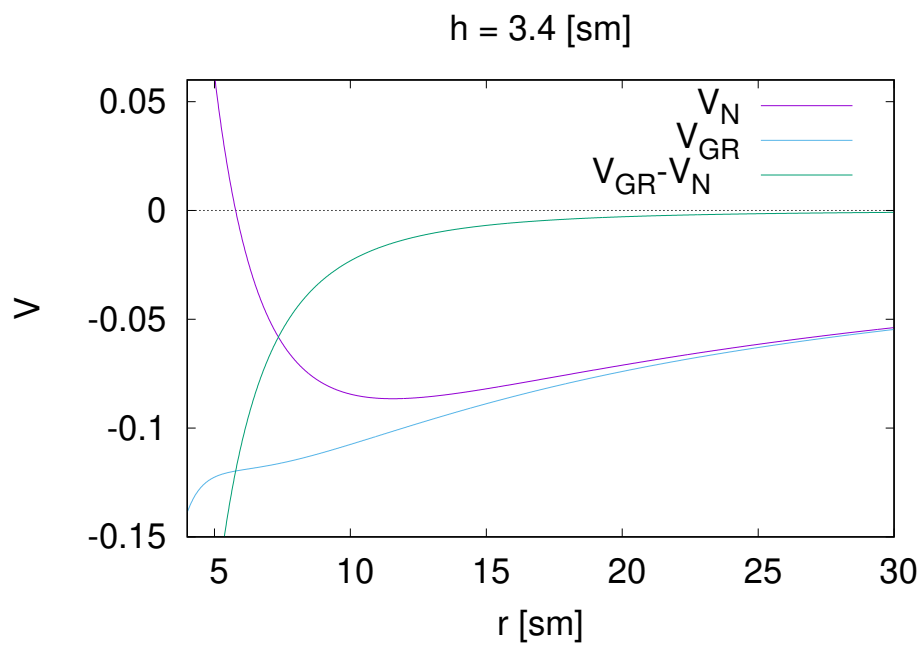
Decreasing $h \leq 4$



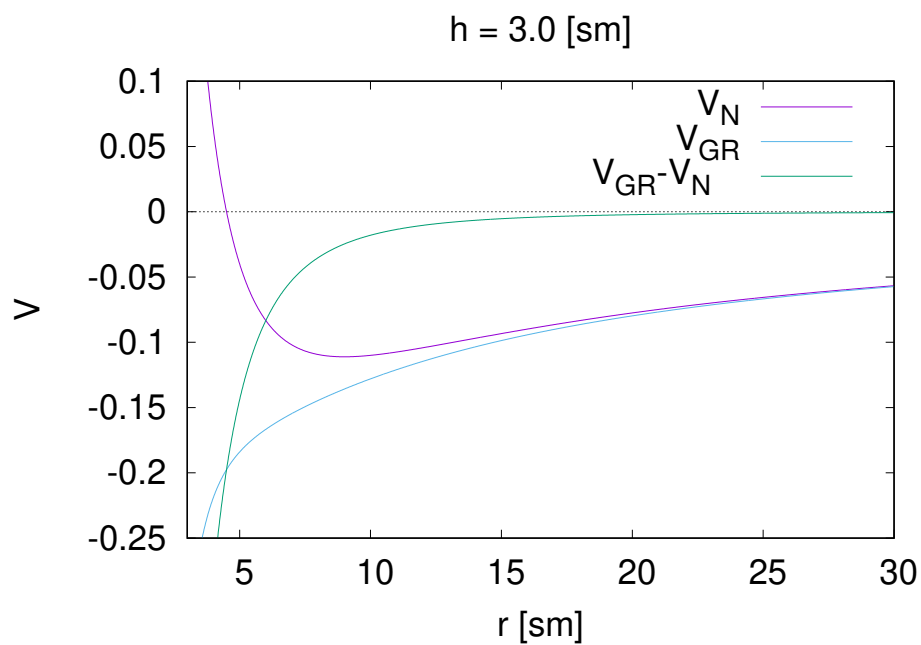
Gravitational Potential.



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