Special and General Relativity PHZ 4601

Test on Homework November 30, 2018.

1. **Twin travel in 2D Minkowski space (36%).**

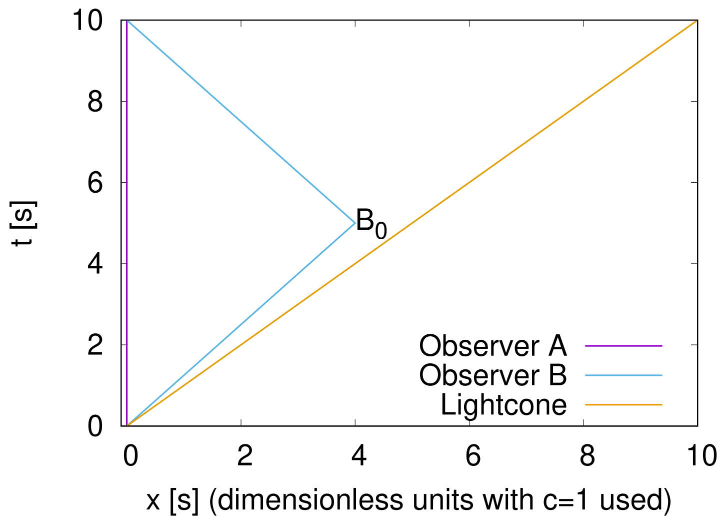

Figure: Minkowski space in which observer B travels away from observer A to the space-time point B_0 and back to A. Natural units with $c = 1$ and arbitrary time units are used.

We use dimensionless units with $c = 1$ in this problem. The figure for this problem is drawn in the rest frame of observer A. The initial position of observers A as well as B is $\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Observer B travels in the rest frame K_1 of A with constant velocity β_B^1 away from A, turns at position B_0 given by $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ and travels back with velocity $-\beta_B^1$ to meet A again. In the following state all results as integers or fractions of integers.

(a) What is the value of β_B^1?

(b) What is the elapsed time on the clock of B at B_0?

(c) What is the elapsed time on the clock of A when A meets B again?

(d) What is the elapsed time on the clock of B when A meets B again?

Consider the inertial frame K_2 in which B is at rest for the first part of its travel, i.e., from the origin to B_0. Arrange K_2 so that its origin agrees at time zero with the origin of K_1.

(e) Find the velocity β_A^2 of A in K_2

(f) Write down the coordinates of B_0 in K_2.

2. **Radar distance (28%).**

The radar distance is defined as $(c/2)\,\triangle\tau$ with $\triangle\tau$ being the proper time elapsed on a standard clock between emission and reception of a radio echo.

Observers at two fixed points A and B in a stationary gravitational field, respectively at Φ_A and Φ_B, determine the radar distance between them by use of standard clocks. Let L_A and L_B be the determinations made at A and B respectively. Find L_A/L_B.

3. **Schwarzschild metric (36%)..**

Note that this problem is for R_{tt} as in HW 32 and not for R_{rr} as in CW 31.

For the diagonal metric

$$d\vec{s}^{\,2} = a\,(dx^1)^2 + b\,(dx^2)^2 + c\,(dx^3)^2 + d\,(dx^4)^2\,,$$

where a, b, c, d are functions of the coordinates, the R_{11} compoment of the Ricci tensor is given by

$$\begin{aligned}
R_{11} \;=\; & + \;\; \beta\,a_{22} + \gamma\,a_{33} + \delta\,a_{44} \\
& + \;\; \beta\,b_{11} + \gamma\,c_{11} + \delta\,d_{11} \\
& - \;\; \beta^2\,b_1^2 - \gamma^2\,c_1^2 - \delta^2\,d_1^2 \\
-\alpha\,a_1 \; (& + \;\; \beta\,b_1 + \gamma\,c_1 + \delta\,d_1 \;\;) \\
-\beta\,a_2 \; (\;\; \alpha\,a_2 & + \;\; \beta\,b_2 - \gamma\,c_2 - \delta\,d_2 \;\;) \\
-\gamma\,a_3 \; (\;\; \alpha\,a_3 & - \;\; \beta\,b_3 + \gamma\,c_3 - \delta\,d_3 \;\;) \\
-\delta\,a_4 \; (\;\; \alpha\,a_4 & - \;\; \beta\,b_4 - \gamma\,c_4 + \delta\,d_4 \;\;)
\end{aligned}$$

where we use the notation

$$\alpha = \frac{1}{2a}\,, \quad \beta = \frac{1}{2b}\,, \quad \gamma = \frac{1}{2c}\,, \quad \delta = \frac{1}{2d}\,,$$

and for $i, j = 1, 2, 3, 4$:

$$a_i = \frac{\partial a}{\partial x^i}\,, \qquad a_{ij} = \frac{\partial^2 a}{\partial x^i\,\partial x^j}\,, \quad \text{and so on.}$$

Consider a stationary, static, spherically symmetric metric of the form

$$d\vec{s}^{\,2} = e^{A(r)}\,dt^2 - e^{B(r)}\,dr^2 - r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right),$$

where we use units with speed of light = 1.

(a) Let $x^1 = t$, $x^2 = r$, $x^3 = \theta$, $x^4 = \phi$ and identify the non-zero contributions to $R_{11} = R_{tt}$.

(b) Use the result of (a) to proof equation (11.4) of the book

$$R_{tt} = -e^{A-B}\left(\frac{A_{22}}{2} - \frac{A_2\,B_2}{4} + \frac{(A_2)^2}{4} + \frac{A_2}{r}\right).$$