## Special and General Relativity PHZ 4601 Midterm October 17 Fall 2018.

## 1. An astrophysical observation ( $40 \%$ ):

For light from some galaxy the spectrum

$$
\lambda=(729.2[n m]) m^{2} /\left(m^{2}-4\right), \quad m=3,4,5, \ldots
$$

is observed. Find the speed at which the galaxy moves away or towards us (ignore the possibility of transverse motion and the expansion of space). Note: In quantum mechanics books you find for the Balmer spectrum of the hydrogen atom $\lambda=(364.56[n m]) m^{2} /\left(m^{2}-4\right)$, and the relativistic Doppler equation reads $\omega^{\prime} / \omega=$ $\sqrt{(1-\beta) /(1+\beta)}$.
2. Light signals and travel in two inertial frames (40\%):


Figure: Minkowski space in which observer A is at rest and flashes a light signal at observer B, who moves with speed $4 / 5$ and flashes the signal back.

In the above figure Minkowski space is parametrized by the coordinates of the rest frame $S$ of an observer A. We use units of seconds $[s]$ and $c=1$ for the speed of light. While observer A stays at rest, observer B moves with speed $\beta=4 / 5$ along the positive $x$ axis. At their common origin both, A and B , have set their clocks to zero. After $15[s]$ observer A emits at position $\mathrm{A}_{1}$, i.e. at ( $15[s], 0$ ), a light signal which reaches observer B at position $\mathrm{B}_{0}$, who flashes it back at observer A , who receives it at position $\mathrm{A}_{2}$ as drawn in the figure.
(a) Find the coordinates of $A_{1}$ in the rest frame $S^{\prime}$ of B , where the translational freedom is used so that the origins of the restframes of A and B agree at time $t=t^{\prime}=0$.
(b) Find the coordinates of $B_{0}$ in the rest frames $S$ of A and $S^{\prime}$ of B.
(c) Find the coordinates of $A_{2}$ in the rest frames $S$ of A and $S^{\prime}$ of B.
(d) Transform the above figure into the rest frame $S^{\prime}$ of B (draw the resulting figure).

## 3. Space travel (20\%):

A spaceship travels with uniform speed to a star 8 light-years away, in a time the crew considers to be 8 years. Find the speed of the spaceship.

