Special and General Relativity PHZ 4601 Fall 2018: Test on Homework Solutions.

1. **Twin travel in 2D Minkowski space.**

   (a) $\beta_B^1 = 4/5$.

   (b) The proper time of B at B_0, the end of the first part of its travel, is

   $$\sqrt{(5)^2 - (4)^2} = 3\,.$$

   (c) $2 \times 5 = 10$ is the time on the clock of A when A meets B again.

   (d) $2 \times 3 = 6$ is the time on the clock of B when A meets B again.

   (e) We have $\beta_A^2 = -\beta_B^1 = -4/5$ for the velocity of A in the inertial frame K_2.

   (f) The coordinates of B_0 in K_2 are $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

2. **Radar Distance.**

   Let the coordinate times for light-signals from A to B and B to A be $\triangle t_{A\to B}$ and $\triangle t_{B\to A}$, respectively. The coordinate round-trip time $\triangle t$ for a radio echo is the same at A and B: $\triangle t = \triangle t_{A\to B} + \triangle t_{B\to A}$. But the radar distance is determined with standard clocks: Radar distance measured at A: $L_A = (c/2)\,\triangle\tau_A = (c/2)\,\exp(\Phi_A/c^2)\,\triangle t$ and, similarly, $L_B = (c/2)\,\triangle\tau_B = (c/2)\,\exp(\Phi_B/c^2)\,\triangle t$. Therefore,

   $$\frac{L_A}{L_B} = \frac{\exp(\Phi_A/c^2)}{\exp(\Phi_B/c^2)}\,.$$

3. **Schwarzschild Metric.**

   We choose the identification

   $$\begin{aligned} d\vec{s}^{\,2} &= a\,(dx^1)^2 + b\,(dx^2)^2 + (dx^3)^2 + (dx^4)^2 \\ &= a\,(dt)^2 + b\,(dr)^2 + c\,(d\theta)^2 + d\,(d\phi)^2\,, \end{aligned}$$

respectively, with $a = \exp[+A(r)]$, $b = -\exp[+B(r)]$, $c = -r^2$, $d = -r^2 \sin^2\theta$. The non-zero derivatives are the

$$' = \frac{\partial}{\partial\, x^2} = \frac{\partial}{\partial\, r} \quad \text{and, on } d \text{ only,} \quad \frac{\partial}{\partial\, x^3} = \frac{\partial}{\partial\, \theta}\,.$$

Therefore, the non-zero contributions to the Ricci tensor are

$$R_{tt} = R_{11} = \beta\, a_{22} - \beta\, a_2 \left(\alpha\, a_2 + \beta\, b_2 - \gamma\, c_2 - \delta_2\, d_2\right),$$

where α, β, γ, δ are defined by $\alpha = 1/2a$, $\beta = 1/2b$, $\gamma = 1/2c$, $\delta = 1/2d$. We have

$$a' = A' \exp(+A)\,, \quad b' = -B' \exp(+B) \quad c' = -2r \quad d' = -2r\,\sin^2\theta$$

and $a'' = A'' \exp(A) + (A')^2 \exp(A)$. Inserting these gives

$$\begin{aligned} R_{tt} &= -\exp(A-B)\,\frac{1}{2}\left(A'' + A'^2\right) \\ &\quad + \exp(A-B)\,\frac{1}{2}\,A'\left(\frac{1}{2}\,A' + \frac{1}{2}\,B' - \frac{1}{r} - \frac{1}{r}\right) \\ &= -\exp(A-B)\left(\frac{1}{2}\,A'' - \frac{1}{4}\,A'\,B' + \frac{1}{4}\,A'^2 + A'/r\right)\,. \end{aligned}$$