Special and General Relativity PHZ 4601 Fall 2018: Midterm Solutions.

1. **An astrophysical observation (redshift):**

The equation for the redshift is

$$\lambda' = \lambda\sqrt{\frac{1+\beta}{1-\beta}} \Rightarrow \left(\frac{\lambda'}{\lambda}\right)^2 = \frac{1+\beta}{1-\beta}$$
$$\left(\frac{\lambda'}{\lambda}\right)^2 - \beta\left(\frac{\lambda'}{\lambda}\right)^2 = 1+\beta$$
$$\left(\frac{\lambda'}{\lambda}\right)^2 - 1 = \beta\left[1+\left(\frac{\lambda'}{\lambda}\right)^2\right] \Rightarrow \beta = \frac{(\lambda'/\lambda)^2-1}{(\lambda'/\lambda)^2+1}\,.$$

With $\lambda' = (729.2\,[nm])\,m^2/(m^2-4)$ given and from Quantum Mechanics books $\lambda = (364.56\,[nm])\,m^2/(m^2-4)$ we find $\lambda'/\lambda = 2$ and, hence,

$$\beta = \frac{v}{c} = \frac{3}{5} = 0.6\,.$$

2. **Light signals and travel in two inertial frames:**

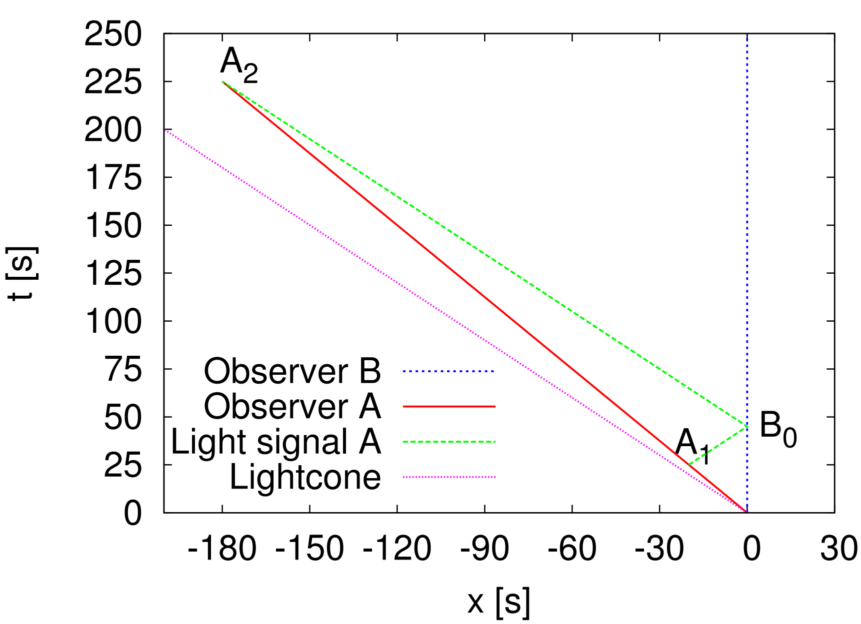

FIG. 1: Minkowski space in which observer B is at rest and observer A moves with speed 4/5 in negative x direction.

We use natural units, $c = 1$, and express everything in seconds, omitting $[s]$ in the following.

(a) In the rest frame S of A the coordinates of A_1 are $(t, x) = (15, 0)$. In the rest frame S' of B we have $15 = \tau = \sqrt{t'^2 - x'^2}$ for the proper time of A at this space-time point and $x' = -4t'/5$. Putting these equations together, we find:

$$15 = \sqrt{(t')^2 - (4t'/5)^2} = 3\,t'/5 \quad \Rightarrow \quad t' = 25 \quad \Rightarrow \quad (t', x') = (25, -20)\,.$$

Alternatively, we may perform the Lorentz transformations

$$t' = \gamma\,t - \beta\gamma\,x \qquad x' = \gamma\,x - \beta\gamma\,t\,.$$

Now, $\beta = 4/5$ and $\gamma = 1/\sqrt{1-(4/5)^2} = 5/3$, $\beta\gamma = 4/3$. Hence, $(t, x) = (15, 0)$ transforms into

$$t' = 5t/3 = 25\,, \quad x' = -4t/3 = -20 \quad \text{i.e.,} \quad (t', x') = (25, -20)\,.$$

(b) In S the coordinates of B_0 are the meeting point of the straight lines $x = 4t/5$ and $x = t - 15$ for $t \geq 15$. The solution is $(t, x) = (75, 60)$ and the proper time of B at this point is $\tau = \sqrt{t^2 - x^2} = 45$. In the S' frame we have by its definition $x' = 0$ for B and the space-time point B_0 becomes $(t', x') = (45, 0)$.

(c) In S the space-time position A_2 is at $(t, x) = (135, 0)$. We can deal with this like before with $(t, x) = (15, 0)$:

$$135 = \sqrt{(t')^2 - (4t'/5)^2} = 3\,t'/5 \quad \Rightarrow \quad t' = 225\,, \quad x' = -4\,t'/5 = -180\,.$$

Alternatively, we may perform the Lorentz transformations. Now,

$$t' = 5t/3 = 225\,, \quad x' = -4t/3 = -180 \quad \text{i.e.,} \quad (t', x') = (225, -180)\,.$$

(d) See the figure.

(e) **Space travel (Rindler problem 2.9):**

$$\triangle x = \beta\,\triangle t = 8\,,$$
$$\triangle\tau = 8 = \frac{\triangle t}{\gamma} = \triangle t\,\sqrt{1-\beta^2} = 8\,\frac{\sqrt{1-\beta^2}}{\beta}\,,$$

Therefore,

$$1 = \frac{\sqrt{1-\beta^2}}{\beta} = \frac{1-\beta^2}{\beta^2} \;\Rightarrow\; 2\,\beta^2 = 1 \;\Rightarrow\; \beta = \frac{1}{\sqrt{2}} = 0.7071\ldots\,.$$