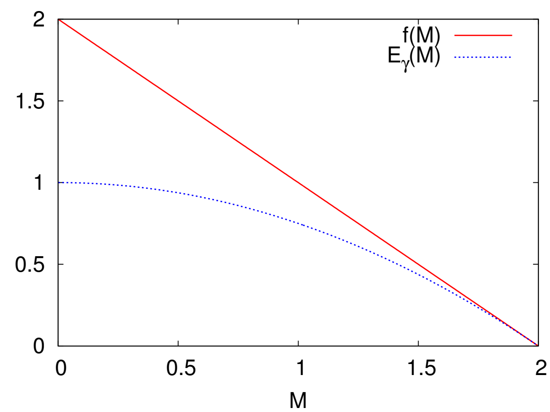
Special and General Relativity (PHZ 4601/5606) Fall 2017 Solutions

Set 5

14. Relativistic Energy-Momentum Conservation.



1. The 4-vector energy-momentum conservation reads

$$\begin{pmatrix} m \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} m \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \sqrt{M^2+p^2} \\ -p \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} E_\gamma \\ p_\gamma \\ 0 \\ 0 \end{pmatrix}$$

Therefore, $p^2 = p_\gamma^2 = E_\gamma^2$ implies

$$2m - E_\gamma = \sqrt{M^2 + E_\gamma^2},,$$

$$(2m - E_\gamma)^2 = 4m^2 - 4m\, E_\gamma + E_\gamma^2 = M^2 + E_\gamma^2\,,$$

$$E_\gamma = \frac{4m^2 - M^2}{4m}\,.$$

2. The requested $E_\gamma(M)$ values are:

$$\begin{aligned} E_\gamma(0) &= 1\,m\,, \\ E_\gamma(m/2) &= \frac{15}{16}\,m\,, \\ E_\gamma(1\,m) &= \frac{3}{4}\,m\,, \\ E_\gamma(\sqrt{2}\,m) &= \frac{1}{2}\,m\,, \\ E_\gamma(\sqrt{3}\,m) &= \frac{1}{4}\,m\,. \end{aligned}$$

3. The sketch is given in the figure.