

Special and General Relativity (PHZ 4601/5606) Fall 2017 Solutions

Set 9

29. Are $t = \text{var}$, $x^i = \text{const}$ lines geodesics?

The metric is $d\bar{s}^2 = dt^2 - dl^2$ and $dx^i = 0$ holds for $t = \text{var}$. Hence, $ds^2 = dt^2 \Rightarrow s = t$. So, $\dot{t} = 1$, $\ddot{t} = 0$ and $\dot{x}^i = 0$, $\ddot{x}^i = 0$.

Now, let us check the geodesic equations of $L = \dot{t}^2 - g_{ij} \dot{x}^i \dot{x}^j$:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{t}} - \frac{\partial L}{\partial t} = 0 \Rightarrow 2\ddot{t} - g_{ij,4} \dot{x}^i \dot{x}^j = 0$$

which is solved by $\ddot{t} = 0$, $\dot{x}^i = 0$. Next,

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}^i} - \frac{\partial L}{\partial x^i} = 0 \Rightarrow -2 \frac{d}{dt} (g_{ij} \dot{x}^j) - g_{jk,i} \dot{x}^j \dot{x}^k = 0.$$

The geodesic equations are satisfied.