Special nd General Relativity (PHZ 4601) Fall 2017

Classwork and Homework

(1) Take as Schwarzschild radius r_s the approximate extension of the visible universe, $r_s = 46 \times 10^9$ $[ly]$.

(a) Calculate calculate the corresponding density in units of $[kg/m^3]$.
(b) How many protons per $[m^3]$ would this be (take the proton weight from Wikipedia)?
(c) Compare with the actual estimated density of the universe (take the estimate of the mass of the universe from Wikipedia, but keep r_s as used above)?
(d) **Solution:** sden.f.

(2) Homework, due F ???/2017 before class.

(a) Find the Shapiro time delay in the approximation of Rindler p.237 for X_1 the distance from Earth to Sun, X_2 the distance from Mercury to Sun and R the radius of the Sun. State the result in seconds. You may take the parameter values from Wikipedia. Calculate to at least three significant digits.
(b) Estimate the distance light travels in the time period which you find.
(c) Repeat the above estimates with Mars replaced by Venus.
(d) **Solution:** shapiro.f.

(3) Homework, due F ???/2017 before class. Assume a circular orbit of radius $r = 149.6 \times 10^9$ $[m]$ for the earth about the sun and use Eq. (11.32) of Rindler to caculate ω in the units $[rad/s]$. Subsequently, calculate the angle covered in one year in the units of $[deg]$. Give at least three significant digits and take the mass of the sun from Google.

(4) Schwarzschild effective potential.

(a) Find the extrema of the Schwarzschild effective potential

$$V_{GR}(r) = \frac{h^2}{r^2} - \frac{2m}{r} - \frac{2m\,h^2}{r^3}$$

in the form

$$r_{1,2} = \frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

where you have to calclate p and q in terms of the central mass m and the angular momentum h.

(b) Express h in terms of m for the solution

$$r_{1,2} = \frac{p}{2} \pm \frac{p}{4}\,.$$

(c) Sketch $V_{GR}(r)$ for the parameters above in the range $0 \leq r \leq 30m$, $-0.08 < V_{GR} < 0.02$ and compare with Newton's effective potential

$$V_N(r) = \frac{h^2}{r^2} - \frac{2m}{r}$$

in the same range. What happens when you increase the values of h in units of m by about 10%?