Special and General Relativity (PHZ 4601/5606) Fall 2017 Solutions Set 2

6. Relativistic distance measurements by light signals.

The elapsed time is 300 $[s]$.

(1) The relation $x = \triangle t/2$ gives for all three times

$$10^{-3} \times 10^5\,[km] \;=\; 300\,[km]\,.$$

Therefore, O_2 is at rest with respect to O_1.

(2) We find the following positions at the following times:

$$\begin{aligned} x_1 &= 300\,[km] \;\text{ at }\; t_1 = 1.001\,[s]\,,\\ x_2 &= 600\,[km] \;\text{ at }\; t_2 = 2.002\,[s]\,,\\ x_3 &= 900\,[km] \;\text{ at }\; t_3 = 3.003\,[s]\,. \end{aligned}$$

This gives the velocities

$$\begin{aligned} v_{21} &= \frac{x_2 - x_1}{t_2 - t_1} \;=\; \frac{300\,[km]}{(2.002 - 1.001\,[s]} \;=\; \frac{300\,[km]}{1.001\,[s]} \;\approx\; 299.7\,[km/s]\,,\\ v_{32} &= \frac{x_3 - x_2}{t_3 - t_2} \;=\; \frac{300\,[km]}{(3.003 - 2.002)\,[s]} \;=\; \frac{300\,[km]}{1.001\,[s]} \;\approx\; 299.7\,[km/s]\,. \end{aligned}$$

So, the results are consistent with the idea that O_2 is at rest in an inertial frame, which moves with about $299.7\,[km/s]$ with respect to the inertial frame of O_1. The position of O_2 with respect to O_1 is then given by

$$x(t) \;=\; x_0 + v\,t \;=\; \frac{300\,[km]}{1.001\,[s]}\,t\,,$$

where $x_0 = 0$ follows from $x(t_1) = 300\,[km]$.

(3) We find again the positions $x_1 = 300\,[km]$ at $t_1 = 1.001\,[s]$ and $x_2 = 600\,[km]$ at $t_2 = 2.002\,[s]$, which gives again $v_{21} = (300/1.001)\,[km/s]$, but now

$$x_3 = 1200\,[km] \;\text{ at }\; t_3 = 3.004\,[s]\,,$$

which gives

$$v_{32} = \frac{x_3 - x_2}{t_3 - t_2} \;=\; \frac{600\,[km]}{1.002\,[s]} \;\approx\; 598.2\,[km/s]\,.$$

As v_{21} and v_{32} disagree, O_2 cannot be at rest in an inertial frame.