Special and General Relativity (PHZ 4601/5606) Fall 2017 Solutions

Set 4

12. Proper time under rotation.

$$c^2\,d\tau^2 = c^2\,dt^2 - \omega^2 R^2\,dt^2 \Rightarrow \frac{d\tau}{dt} = \sqrt{1 - \frac{\omega^2 R^2}{c^2}}\ .$$

Now $g = v^2/R = \omega^2\,R$, where $g = 9.81\,[m/s^2]$ and v is the velocity of the satellite. For $R = (6,378 + 160)\,[km]$ (equatorial radius plus $160\,[km]$) we get $\omega = 0.0012249\,[s^{-1}]$, which correpsonds to a period of $T = 2\pi/\omega = 85.5\,[minutes]$. For the time dilation we find

$$1 - \frac{d\tau}{dt} = \frac{1}{2}\left(\frac{\omega\,R}{c}\right)^2 = 0.356321 \times 10^{-9}\,.$$

Compared to this the time dilation of a clock on the equator is with

$$1 - \frac{d\tau}{dt} = \frac{1}{2}\left(\frac{\omega\,R}{c}\right)^2 = 0.119517 \times 10^{-11}$$

negligle. For gravity we obtain along the lines of the previous problem 4

$$1 - \frac{d\tau}{dt} = -\,0.17422 \times 10^{-10}\,,$$

which reduces the the SR effect by about 5%.