Special and General Relativity (PHZ 4601/5606) Fall 2017

Set 6

Relative velocity: Assume that S' moves with velocity $\beta\,\hat{x}$ with respect to a frame S. What is the velocity $\vec{\beta}_2'$ then in S? According to the earlier problem 13 we have:

$$\beta_2^1 = \frac{\beta_2'^1 + \beta}{1 + \beta_2'^1\,\beta} \quad \text{and} \quad \beta_2^2 = \frac{\beta_2'^2}{\gamma\,(1 + \beta_2'^1\,\beta)}\,.$$

To have the first particle at rest in S we have to set $\beta = -\beta_1'^1 = -1/2$. As $\beta_2'^1 = 0$ we get

$$\beta_2^1 = \beta = -1/2 \quad \text{and} \quad \beta_2^2 = \frac{\beta_2'^2}{\gamma} = \frac{1/2}{\gamma}\,.$$

Now,

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-1/4}} = \frac{2}{\sqrt{3}} \quad \Rightarrow \quad \beta_2^2 = \frac{\sqrt{3}}{4}\,.$$

For the length we have

$$\left|\vec{\beta}_2\right| = \sqrt{(\beta_2^1)^2 + (\beta_2^2)^2} = \sqrt{\frac{1}{4} + \frac{3}{16}} = \sqrt{\frac{1}{4} + \frac{3}{16}} = \sqrt{\frac{4+3}{16}} = \frac{\sqrt{7}}{4}\,.$$