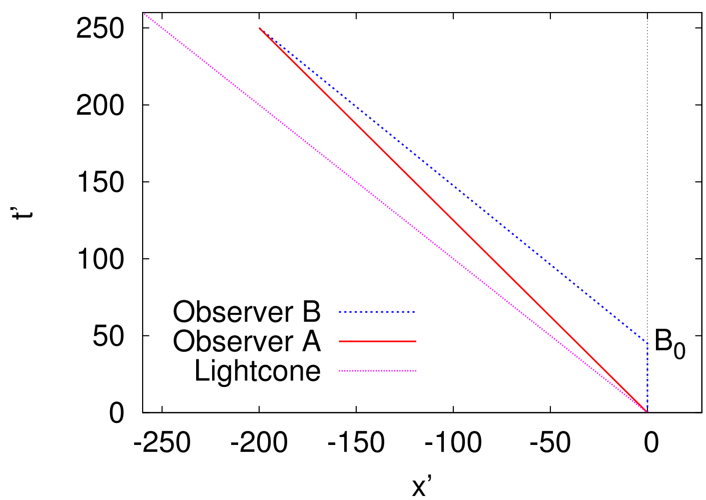
Special and General Relativity (PHZ 4601/5606) Fall 2017 Solutions

Set 7

22. Twin travel in 2D Minkowski space.



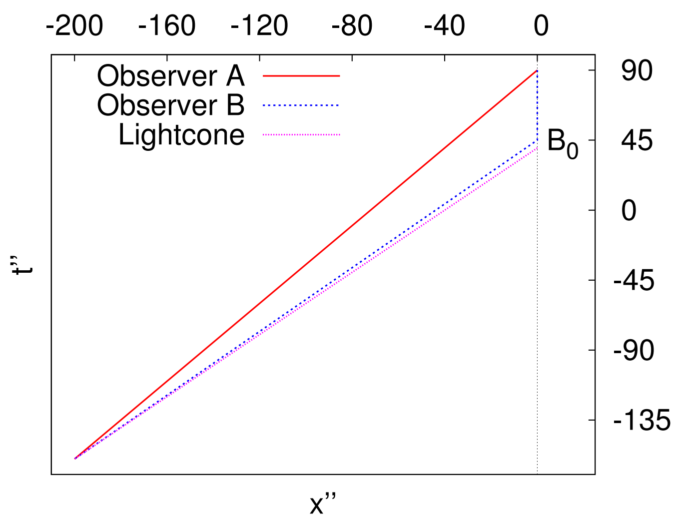
IRS: Minkowski space in which observer B is at rest and observer A moves with speed $4c/5$ in negative x' direction.

(1) $\beta_1 = 60/75 = 4/5$.

(2) The proper time of B at the end of the first part of its travel is $\sqrt{(75)^2-(60)^2} = 15\sqrt{5^2-4^2} = 45$. This is also the time in the initial rest frame at which B leaves the $x=0$ axis.

(3) There are several ways to calculate the final meeting point. In the K frame the final position of B is $\begin{pmatrix}150\\0\end{pmatrix}$. For the Lorentz transformation to this IRF K' we have to use

$$\gamma = \sqrt{\frac{1}{1-\beta^2}} = \sqrt{\frac{5^2}{5^2-4^2}} = \frac{5}{3} \text{ and } \beta\gamma = \frac{4}{3}\,.$$



FRS: Minkowski space in which observer B is at rest and observer A moves with speed $4c/5$ in positive x'' direction.

So, we find for the position of the final point in K'

$$\begin{pmatrix} x'^0 \\ x'^1 \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \end{pmatrix} = \begin{pmatrix} 5/3 & -4/3 \\ -4/3 & 5/3 \end{pmatrix} \begin{pmatrix} 150 \\ 0 \end{pmatrix} = \begin{pmatrix} 250 \\ -200 \end{pmatrix} .$$

See the figure.

Note: $\sqrt{(250-45)^2 - 200^2} = 5\sqrt{41^2 - 40^2} = 45$ is the proper time for the second part of the travel of B.

(4) From the previous result we see that B travels with the velocity

$$\beta' \;=\; -\frac{200}{205} \;=\; -\frac{40}{41}$$

to catch up with A, B. The same result follows from the addition theorem of velocities:

$$-\beta' \;=\; -\frac{\beta+\beta}{1+(\beta)^2} \;=\; -\frac{8}{5}\,\frac{25}{41} \;=\; -\frac{40}{41}\,.$$

Calculating the velocity first, we coul have calculated the coordinates of the last position of B in K' as the meeting point of the tracetories $x = -4\,t/5$ and $x = 40\,(t-45)/41$.

Note: See the second figure for the same in the final rest frame (FRS).