

## Solution Problem 2

Find  $\begin{pmatrix} x \\ y \end{pmatrix}$  in  $K$  from  $\begin{pmatrix} x' \\ y' \end{pmatrix}$  in  $K'$ .

$$\begin{aligned}\hat{x}' &= (\hat{x}' \cdot \hat{x}) \hat{x} + (\hat{x}' \cdot \hat{y}) \hat{y} \\ &= \cos \phi \hat{x} + \sin \phi \hat{y}\end{aligned}$$

$$\begin{aligned}\hat{y}' &= (\hat{y}' \cdot \hat{x}) \hat{x} + (\hat{y}' \cdot \hat{y}) \hat{y} \\ &= -\sin \phi \hat{x} + \cos \phi \hat{y}\end{aligned}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{aligned} &x' \cos \phi \hat{x} - y' \sin \phi \hat{x} \\ &+ x' \sin \phi \hat{y} + y' \cos \phi \hat{y} \end{aligned}$$

$$= \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$= M(\phi) \begin{pmatrix} x' \\ y' \end{pmatrix}$$

Note:  $M(-\phi) M(\phi) = 1$

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Isomorphism:  $1 \leftrightarrow 1$