

Special and General Relativity (PHZ 4601/5606) Fall 2017

Set 6

20. Dual Tensor.

From $*E^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}E_{\gamma\delta}$ we obtain for the dual tensor

$$*E^{12} = E_{34}, \quad *E^{13} = -E_{24}, \quad *E^{14} = E_{23},$$

$$*E^{23} = E_{14}, \quad *E^{24} = -E_{13}, \quad *E^{34} = E_{12}.$$

In matrix form the dual tensor reads then

$$(*E^{\alpha\beta}) = \begin{pmatrix} 0 & *E^{12} & *E^{13} & *E^{14} \\ *E^{21} & 0 & *E^{23} & *E^{24} \\ *E^{31} & *E^{32} & 0 & *E^{34} \\ *E^{41} & *E^{42} & *E^{43} & 0 \end{pmatrix} = \begin{pmatrix} 0 & E_{34} & -E_{24} & E_{23} \\ -E_{34} & 0 & E_{14} & -E_{13} \\ E_{24} & -E_{14} & 0 & E_{12} \\ -E_{23} & E_{13} & -E_{12} & 0 \end{pmatrix}.$$

Using the $E_{\mu\nu}$ equations of (7.32) this becomes

$$(*E^{\alpha\beta}) = \begin{pmatrix} 0 & -e_3 & e_2 & -b_1 \\ e_3 & 0 & -e_1 & -b_2 \\ -e_2 & e_1 & 0 & -b_3 \\ b_1 & b_2 & b_3 & 0 \end{pmatrix}.$$

The equation $*E^{\alpha 4}_{,\alpha} = 0$ is the homogeneous Maxwell equation

$$\nabla \cdot \vec{b} = 0.$$

The other three equations $*E^{\alpha i}_{,\alpha} = 0$, $i = 1, 2, 3$ yield the homogeneous Maxwell equation

$$\nabla \times \vec{e} + \frac{1}{c} \frac{\partial \vec{b}}{\partial t} = 0.$$