

Special and General Relativity (PHZ 4601/5606) Fall 2017 Solutions
Set 3

9. Relativistic distance measurements by light signals.

The elapsed time is 300 [s] .

- (1) The relation $x = \Delta t/2$ gives for all three times

$$10^{-3} \times 10^5 \text{ [km]} = 300 \text{ [km]}.$$

Therefore, O_2 is at rest with respect to O_1 .

- (2) We find the following positions at the following times:

$$x_1 = 300 \text{ [km]} \text{ at } t_1 = 1.001 \text{ [s]},$$

$$x_2 = 600 \text{ [km]} \text{ at } t_2 = 2.002 \text{ [s]},$$

$$x_3 = 900 \text{ [km]} \text{ at } t_3 = 3.003 \text{ [s]}.$$

This gives the velocities

$$v_{21} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{300 \text{ [km]}}{(2.002 - 1.001) \text{ [s]}} = \frac{300 \text{ [km]}}{1.001 \text{ [s]}} \approx 299.7 \text{ [km/s]},$$

$$v_{32} = \frac{x_3 - x_2}{t_3 - t_2} = \frac{300 \text{ [km]}}{(3.003 - 2.002) \text{ [s]}} = \frac{300 \text{ [km]}}{1.001 \text{ [s]}} \approx 299.7 \text{ [km/s]}.$$

So, the results are consistent with the idea that O_2 is at rest in an inertial frame, which moves with about 299.7 [km/s] with respect to the inertial frame of O_1 . The position of O_2 with respect to O_1 is then given by

$$x(t) = x_0 + vt = \frac{300 \text{ [km]}}{1.001 \text{ [s]}} t,$$

where $x_0 = 0$ follows from $x(t_1) = 300 \text{ [km]}$.

- (3) We find again the positions $x_1 = 300 \text{ [km]}$ at $t_1 = 1.001 \text{ [s]}$ and $x_2 = 600 \text{ [km]}$ at $t_2 = 2.002 \text{ [s]}$, which gives again $v_{21} = (300/1.001) \text{ [km/s]}$, but now

$$x_3 = 1200 \text{ [km]} \text{ at } t_3 = 3.004 \text{ [s]},$$

which gives

$$v_{32} = \frac{x_3 - x_2}{t_3 - t_2} = \frac{600 \text{ [km]}}{1.002 \text{ [s]}} \approx 598.2 \text{ [km/s]}.$$

As v_{21} and v_{32} disagree, O_2 cannot be at rest in an inertial frame.