Special and General Relativity (PHZ 4601/5606) Fall 2017 Solutions

Set 7

23. Distance in a saddle.

For $y = 0$ we have $z = x^2$ and, therefore, along the curve

$$ds^2 = df^2 + dx^2 = (2x\,dx)^2 + dx^2 = (4x^2 + 1)\,dx^2 \,.$$

With $ds' = \sqrt{4x'^2 + 1}\,dx'$ we integrate then

$$s = \int_0^s ds' = \int_0^x \sqrt{4x'^2 + 1}\,dx' = \frac{x}{2}\sqrt{4x^1 + 1} + \frac{1}{4}\ln\left(\sqrt{4x^2 + 1} + 2x\right) \,,$$

where the integration was done with the online integral calculator from Mathematica. By bisection of the function $s(x)$ we obtain for $s = 10$ the value

$$x = 3.041 \,.$$