Special and General Relativity (PHZ 4601/5606) Fall 2017 Solutions

Set 12

39. Lorenz gauge with covariant derivatives. Exercise 14.1 of Rindler.

According to (10.55) of Rindler we have

$$\Phi^{\mu}{}_{;\rho\sigma} - \Phi^{\mu}{}_{;\sigma\rho} = -\Phi^{\alpha}\, R^{\mu}{}_{\alpha\rho\sigma}\,.$$

Therefore, the definition (10.68) of the Ricci tensor yields

$$\Phi^{\mu}{}_{;\rho\mu} - \Phi^{\mu}{}_{;\mu\rho} = -\Phi^{\alpha}\, R^{\mu}{}_{\alpha\rho\mu} = -\Phi^{\alpha}\, R_{\alpha\rho}\,.$$

In the Lorenz gauge $\Phi^{\mu}{}_{;\mu\rho} = 0$ holds and we are left with

$$\Phi^{\mu}{}_{;\rho\mu} = -\Phi^{\alpha}\, R_{\alpha\rho} \quad \text{or} \quad \Phi^{\mu}{}_{;}{}^{\rho}{}_{\mu} = -\Phi^{\alpha}\, R_{\alpha}{}^{\rho}\,.$$

Renaming the dummy indices ν and the free index $\rho \to \mu$ we obtain the desired result

$$\Phi^{\nu}{}_{;}{}^{\mu}{}_{\nu} = -\Phi^{\nu}\, R_{\nu}{}^{\mu}\,.$$

As the Ricci tensor is symmetric, we can as well write R^{μ}_{ν}.