Special and General Relativity (PHZ 4601/5606) Fall 2017 Solutions

Set 5

15. Spacetrip Elapsed time on earth.

We only consider the first quarter (five years) of the flight. The other results for t are the same by symmetry reasons. With $\beta = v/c$ the acceleration in the rest frame is given by

$$\frac{g}{c} = \frac{d\beta}{d\tau} = \frac{d\zeta}{d\tau}\,,$$

where ζ is the rapidity. As rapidities are additive

$$d\zeta(\tau) = \frac{d\zeta}{d\tau}\,d\tau = \frac{g}{c}\,d\tau$$

holds. With the initial condition $\zeta(0) = 0$ this integrates to

$$\zeta = \int_0^{\zeta} d\zeta' = \frac{g}{c}\int_0^{\tau} d\tau' = \frac{g}{c}\,\tau\,.$$

The age of the twin on earth follows from $d\tau = dt/\cosh(\zeta)$:

$$\int_0^t dt' = t = \int_0^{\tau} \cosh[\zeta(\tau')]\,d\tau' = \int_0^{\tau} \cosh\left[\frac{g}{c}\,\tau'\right]\,d\tau' = \frac{c}{g}\,\sinh\left[\frac{g}{c}\,\tau\right]\,.$$

Inserting $\tau = 5\,\text{years} = 5 \times 365 \times 24 \times 3600\,[s]$, $c = 3 \times 10^8\,[m/s]$ and $g = 9.81\,[m/s^2]$, we find $t = 84.1$ years for a quarter of the trip and 337 years for the entire trip.