Special and General Relativity (PHZ 4601/560 Fall 2017)

Test on Homework

1. Time dilation for a satellite from special relativity (neglect gravitational effects) (20%).

   The circular orbit of a satellite is described by its radius R and angular velocity ω. Use special relativity to calculate the ratio of the proper time interval $d\tau$ of a clock on the satellite over the time interval dt of the inertial frame in which the center of the orbit is at rest.

   Use $d\tau$ for a clock on the satellite and dt for a clock fixed on the equator of the earth. Calculate the numerical value $1 - d\tau/dt$ for a satellite that orbits above the equator at an altitude of 160 [km]. Use $g = 9.81\,[m/s^2]$ for the gravitational acceleration and $R = 6378\,[km]$ for the radius of the earth.

2. Flat 2D plane mapped by the standard polar coordinates, $x = r\,\cos\theta$, $y = r\,\sin\theta$ (60%).

   (a) Write down ds^2.

   (b) What are the coefficients g_{ij}?

   (c) What are the coefficients g^{ij}?

   (d) Calculate the Christoffel symbols $\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}\,g^{\lambda\gamma}\,(g_{\gamma\mu,\nu} + g_{\gamma\nu,\mu} - g_{\mu\nu,\gamma})$ for this system.

   (e) Using

   $$\ddot{x}^{\lambda} + \Gamma^{\lambda}_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} = 0$$

   derive the two coupled, second-order differential equations that describe the motion of a free particle in this system.

3. Is $t = var$, x_i fixed, geodesic? (20%).

   Consider the spacetime metric $d\vec{s}^{\,2} = dt^2 - dl^2$, where dl^2 is an arbitrary and possibly time-dependent metric. Are coordinate lines with t variable and the other coordinates fixed at constant values geodesic? Hint: Investigate whether the geodesic equations of the Lagrangian

   $$L = \dot{t}^{\,2} - g_{ij}\,\dot{x}^i\,\dot{x}^j$$

   allow for the proposed solutions.