Special and General Relativity (PHZ 4601/560 Fall 2017)

Solutions Test on Homework November 20.

1. Proper time under rotation.

$$c^2\,d\tau^2 = c^2\,dt^2 - \omega^2 R^2\,dt^2 \Rightarrow \frac{d\tau}{dt} = \sqrt{1-\frac{\omega^2 R^2}{c^2}} \approx 1 - \frac{1}{2}\,\frac{\omega^2\,R^2}{c^2}\ .$$

Now $g = v^2/R = \omega^2\,R$, where $g = 9.81\,[m/s^2]$ and v is the velocity of the satellite. For $R = (6,378+160)\,[km]$ (equatorial radius plus $160\,[km]$) we get $\omega = 0.0012249\,[s^{-1}]$, which correpsonds to a period of $T = 2\pi/\omega = 85.5\,[minutes]$. For the time dilation we find

$$1-\frac{d\tau}{dt} = \frac{1}{2}\left(\frac{\omega\,R}{c}\right)^2 = 0.356321\times 10^{-9}\,.$$

Compared to this the time dilation of a clock on the equator is negligible:

$$1-\frac{d\tau}{dt} = \frac{1}{2}\left(\frac{\omega\,R}{c}\right)^2 = 0.119517\times 10^{-11}\,.$$

2. See sol26.pdf of the Homework, Set 8.

3. Are $t = var$, $x^i = const$ lines geodesics?

The metric is $d\vec{s}^{\,2} = dt^2 - dl^2$ and $dx^i = 0$ holds for $t = var$. Hence, $ds^2 = dt^2 \Rightarrow s = t$. So, $\dot{t} = 1$, $\ddot{t} = 0$ and $\dot{x}^i = 0$, $\ddot{x}^i = 0$.

Now, let us check the geodesic equations of $L = \dot{t}^2 - g_{ij}\,\dot{x}^i\,\dot{x}^j$:

$$\frac{d}{dt}\,\frac{\partial L}{\partial\,\dot{t}} - \frac{\partial L}{\partial\,t} = 0 \quad\Rightarrow\quad 2\,\ddot{t} - g_{ij,4}\,\dot{x}^i\,\dot{x}^j = 0$$

which is solved by $\ddot{t} = 0$, $\dot{x}^i = 0$. Next,

$$\frac{d}{dt}\,\frac{\partial L}{\partial\,\dot{x}^i} - \frac{\partial L}{\partial\,x^i} = 0 \quad\Rightarrow\quad -\,2\,\frac{d}{dt}\left(g_{ij}\,\dot{x}^j\right) - g_{jk,i}\,\dot{x}^j\,\dot{x}^k = 0\,.$$

The geodesic equations are satisfied.