Special and General Relativity (PHZ 4601/5606) Fall 2018
Classwork and Homework

Every exercise counts 10 points unless stated differently.

Set 1:

(1) Homework, due F 8/31/2018 before class. Consider the 2D Euclidean vectors $\vec{r}_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$ and $\vec{r}_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$. Prove or disprove algebraically that their scalar product $\vec{r}_0 \cdot \vec{r}_1$ is invariant under a rotation by an angle ϕ.

(2) Homework, due F 8/31/2018 before class, 5 points. Use $c = 3 \times 10^5$ $[km/s]$ and $g = 9.81$ $[m/s^2]$, and one year = 365 days to calculate g in units of light years $[ly]$ and years $[y]$ to three significant digits.

(3) Homework, due F 8/31/2018 before class, 5 points. Consider the triangle of Fig. 8.4 (p.168) of the book for the special case $r = a\,\pi/2$ and show that equation (1.9) holds.

Set 2:

(4) Homework, due F 9/7/2018 before class. Exercise 1.6 of Rindler.

Set 3:

(5) Homework, due F 9/14/2018 before class. Use $c = 3 \times 10^5$ $[km/s]$, $g = 9.8$ $[m/s^2]$, $[ft] = 0.3[m]$, and one year = 365 days. Estimate the time difference encountered due to the gravitational frequency shift after running for one year identical atomic clocks positioned in Colorado at an altitude of $5400\,[ft]$ and at the Greenwich Observatory at an altitude of $80\,[ft]$. State the result in microsecond and round to the first two digits. Assume that g is constant in the range from $80\,[ft]$ to $5400\,[ft]$ and give a reason why that is a reasonable approximation.

(6) Homework, due F 9/14/2018 before class. Exercise 1.9 of Rindler. By "source" a light source is meant.

(7) Classwork, due F 9/12/2018 in class: Time and relativistic distance measurements (synchronization of clocks).

A Cesium clock counts 2,757,789,531,312 cycles. Find the elapsed time in seconds and round to the nearest integer number.

In the following approximate the speed of light by $300,000\,[km/s]$, but perform all calculations with a precision of at least four digits.

An observer O_1 is located in an inertial system and flashes at times $1,\,2,\,3\,\ldots\,[s]$ light signals towards another observer O_2, who reflects them back with a mirror. Assume that the returned signals are received by O_1 at the following times:

(1) 1.002, 2.002, 3.002 $\ldots\,[s]$,

(2) 1.002, 2.004, 3.006 $\ldots\,[s]$,

(3) 1.002, 2.004, 3.008 $\ldots\,[s]$.

Determine for each case whether the data are consistent with assuming that a frame attached to O_2 is also an inertial system. If this is the case, write down the equation for the distance of O_2 as function of the time as seen by O_1, $x(t) = x(0) + v\,t$.

Set 4:

(8) Homework, due F 9/21/2017 before class: Euclidean and hyperbolic rotations.

Consider the 2D Euclidean rotation

$$\begin{pmatrix} x'^{\,1} \\ x'^{\,4} \end{pmatrix} = \begin{pmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{pmatrix} \begin{pmatrix} x^1 \\ x^4 \end{pmatrix}$$

and substitute $\phi = i\,\zeta$, $x^4 = i\,x^0$, $x'^{\,4} = i\,x'^{\,0}$. Write out the equations for $x'^{\,1}$ and $x'^{\,0}$ in terms of sinh and cosh.

(9) Homework, due F 9/21/2018. Twin travel in 2D Minkowski space.

We use dimensionless units with $c = 1$ in this problem. The figure for this problem is drawn in the rest frame of observer A. The initial position of observers A as well as B is $\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Observer B travels in the rest frame K_1 of A with constant velocity β^1_B away from A, turns at position B_0 given by $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ and travels back with velocity $-\beta^1_B$

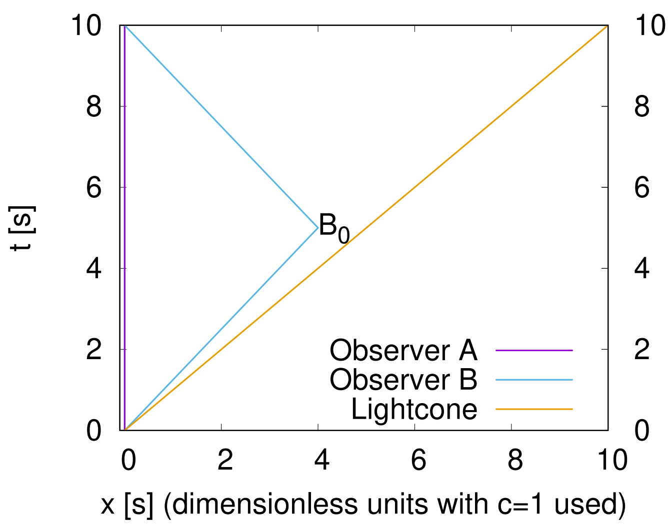

Figure: Minkowski space in which observer B travels away from observer A to the space-time point B_0 and back to A. Natural units with $c = 1$ and arbitrary time units are used.

to meet A again. In the following state all results as integers or fractions of integers.

(a) What is the value of β_B^1?
(b) What is the elapsed time on the clock of B at B_0?
(c) What is the elapsed time on the clock of A when A meets B again?
(d) What is the elapsed time on the clock of B when A meets B again?

Consider the inertial frame K_2 in which B is at rest for the first part of its travel, i.e., from the origin to B_0. Arrange K_2 so that its origin agrees at time zero with the origin of K_1.

(e) Find the velocity β_A^2 of A in K_2
(f) Write down the coordinates of B_0 in K_2.
(g) What is the velocity β_B^2 in K_2 of B from B_0 to its final position, where A meets B again.
(h) What are the coordinates in K_2 of the position where A meets B again?
(i) Draw the travel in K_2 similarly as it is drawn for K_1 in the figure.

(10) Homework, due F 9/21/2017 before class. Time dilation for a satellite.

For this problem use $G = 6.674 \times 10^{-11}\,[m^3\,kg^{-1}\,s^{-2}]$ for Newton's gravitational constant, $m_e = 5.972 \times 10^{24}\,[kg]$ for the mass of the earth and $r_e = 6371 \times 10^3\,[m]$ for the radius of the earth. The earth is approximated by a sphere.

Use special relativity to calculate the ratio of the proper time interval $d\tau$ of a clock on the satellite over the time interval dt of the inertial frame in which the center of the orbit is at rest.

A stationary satellite orbits on a circle about the center of the earth. Find the radius of this circle and compare with the radius of the earth (given above).

Use $d\tau_s$ for a clock on the stationary satellite and $d\tau_e$ for a clock fixed on the equator of the earth. Calculate the numerical value $1 - d\tau_s/d\tau_e$ to order c^{-2}.

Calculate the the competing effect from gravity for the same satellite.

Set 5:

(11) Homework, due F 9/28/2018. Twin travel in 2D Minkowski space. Continuation of the previous problem.
Consider the inertial frame K_3 in which B is for the second part of its travel, i.e., from B_0 to the position where A meets B again, at rest. Arrange K_3 so that its origin agrees at time zero with the origin of K_1. State again all results as integers or fractions of integers.

(a) Find the velocity β^3_A of A in K_3
(b) What is the velocity β^3_B of B in K_3 from the origin to B_0.
(c) Write down the coordinates of B_0 in K_3.
(d) What is the elapsed time on the clock of B at B_0?
(e) What are the coordinates in K_3 of the position where A meets B again?
(f) Draw the travel in K_3 similarly as it is drawn for K_1 in the figure of the previous problem.
(g) Write down the translation that transforms the inertial frame K_3 into the inertial frame K_4 for which the B_0 position agrees with the B_0 position in inertial frame K_2.
(h) Draw the travel in K_4.

(i) Patch the figures for K_2 and K_4 together in the following way: Take the worldlines of K_2 up to the time of B_0 in K_2. For times larger than the time of B_0 in K_2 take the worldlines of K_4. Plot the resulting figure.
(j) If you did everything right, the worldlines of A from K_2 and K_4 meet at $t = 3$, $x = -2.4$ in the last figure. Are these identical space-time points? Find this space-time point from K_2, then from K_4, in the initial frame K_1.

(12) Homework, due F 9/28/2017 before class. Spacetrip and elapsed time on Earth (adapted from Jackson, Electrodynamics).

Assume that the earth is in an inertial frame. A spaceship leaves the earth in the year zero. The spaceship is constructed so that it has an acceleration g in its own frame (to make the occupants feel comfortable). By its own clock, it accelerates on a straight-line path for 5 years, decelerates at the same rate for 5 more years, turns around, accelerates for 5 years, decelerates for 5 years, and lands on earth. What is the time on earth?

How far did the spaceship travel?

Instructions: Use $g = 9.81\ [m/s^2]$, one year $= 365 \times 24 \times 3600\ [s]$, and for the speed of light $c = 300,000\ [km/s]$. Calculate three significant digits. Hint: Find first $\zeta(\tau)$, where τ is the proper time in the rocket and $\zeta(\tau)$ its rapidity with respect to Earth.

(13) Homework, due F 9/29/2017 before class.

Two particles, each of mass m and with negligible kinetic energy, annihilate into a particle of mass M and a photon.
1. Use natural units with $c = 1$ and calculate the photon energy $E_\gamma = |p_\gamma|$ as function of M in the range $0 \le M < 2m$.
2. Compute $E_\gamma(M)$ for $M = 0,\ 0.5\,m,\ 1\,m,\ \sqrt{2}\,m,\ \sqrt{3}\,m.$
3. Sketch $f(M) = 2m - M$ and $E_\gamma(M)$ in one graph together.

Set 6:

(14) Homework, due F 10/5/2018 before class. End of spacetrip.

Assume that the spaceship of the previous exercise moves by exhausting particles (photons) at the speed of light c.

1. Derive an expression for $m(\tau)$, the (remaining) mass of the spaceship at proper time τ.

2. Which fraction of the original mass is left, after the spacetrip has been performed?

(15) Homework, due F 10/5/2018 before class. Ccontractions.
Let (in unspecified units)

$$\begin{aligned}
&(1)\ \ ct = 5,\ x^1 = 1,\ x^2 = 2,\ x^3 = 3,\\
&(2)\ \ ct = 5,\ x_1 = 1,\ x^2 = 2,\ x^3 = 3,\\
&(3)\ \ ct = 5,\ x^1 = 1,\ x^2 = 2,\ x^3 = -3,\\
&(4)\ \ ct = 5,\ x^1 = 0,\ x^2 = 3,\ x^3 = 4,\\
&(5)\ \ ct = 5,\ x_1 = 0,\ x_2 = 3,\ x_3 = 4,\\
&(6)\ \ ct = 5,\ x^1 = 2,\ x^2 = 3,\ x^3 = 4,\\
&(7)\ \ ct = 5,\ x^1 = 0,\ x^2 = 3,\ x^3 = -4.
\end{aligned}$$

Calculate $x_\alpha x^\alpha$ for each case. Comment on $x^\alpha\, x_\alpha$.

(16) Homework, due F 10/5/2017 before class. Levi-Civita tensor.
The four-dimensional Levi-Civita tensor $\epsilon^{\alpha\beta\gamma\delta}$ is completely antisymmetric in its indices and $\epsilon^{1234} = 1$. Show the following properties:

(1) Show $\epsilon_{\alpha\beta\gamma\delta} = -\epsilon^{\alpha\beta\gamma\delta}$.

(2) Express $\epsilon_{\alpha\beta\gamma_1\delta_1}\epsilon^{\alpha\beta\gamma_2\delta_2}$ in terms of Kronecker delta.

(3) Express $\epsilon_{\alpha\beta_1\gamma_1\delta_1}\epsilon^{\alpha\beta_2\gamma_2\delta_2}$ in terms of Kronecker delta.

(17) Classwork, due 10/3/2018 in class. Electromagnetic field tensor.
Let $E^{\alpha\beta}$ be an antisymmetric tensor of rank 2. Compare

$$\frac{\partial E^{\alpha\beta}}{\partial\, x^\alpha} = E^{\alpha\beta}{}_{,\alpha} = \frac{4\pi}{c}\, J^\beta$$

for $\beta = 4$ with the inhomogeneous Maxwell equations

$$\nabla\cdot\vec{e} = 4\pi\rho = \frac{4\pi}{c}\, J^4, \quad \nabla\times\vec{b} - \frac{1}{c}\frac{\partial\vec{e}}{\partial t} = \frac{4\pi}{c}\vec{J}$$

and identify E^{44}, E^{14}, E^{24} and E^{34} in terms of $\vec{e}$ and $\vec{b}$. Continue with $\beta = 1$, then $\beta = 2$. Do you need also $\beta = 3$ to determine all components of the tensor $E^{\alpha\beta}$ in terms of $\vec{e}$ and $\vec{b}$?

Write down the matrix of the finally obtained electromagnetic field tensor $\left(E^{\alpha\beta}\right)$ in terms of $\vec{e}$ and $\vec{b}$.

(18) Classwork, 5 points, due F 10/5/2018 in class.
Raising, lowering and contractions of indices.

Let $T^{\alpha\beta}$ be an arbitrary SR rank 2 tensor (T for tensor) .

(a) Circle the correct equations in the following.

$$\begin{aligned}
T^{11} &= +T_{11} \quad \text{or} \quad T^{11} = -T_{11}\,, \\
T^{12} &= +T_{12} \quad \text{or} \quad T^{12} = -T_{12}\,, \\
T^{13} &= +T_{13} \quad \text{or} \quad T^{13} = -T_{13}\,, \\
T^{14} &= +T_{14} \quad \text{or} \quad T^{14} = -T_{14}\,, \\
T^{41} &= +T_{41} \quad \text{or} \quad T^{41} = -T_{41}\,, \\
T^{42} &= +T_{42} \quad \text{or} \quad T^{42} = -T_{42}\,, \\
T^{43} &= +T_{43} \quad \text{or} \quad T^{43} = -T_{43}\,, \\
T^{44} &= +T_{44} \quad \text{or} \quad T^{44} = -T_{44}\,,
\end{aligned}$$

(b) Write out the 16 terms of $T^{\alpha\beta}\,T_{\alpha\beta}$.

Set 7:

(19) Homework, due M 10/15/2018 before class.
The dual tensor is defined by

$$^*E^{\alpha\beta} = \frac{1}{2}\,\epsilon^{\alpha\beta\gamma\delta}E_{\gamma\delta}\,.$$

Write down the matrix $\left({}^*E^{\alpha\beta}\right)$ and express it subsequently in terms of $E_{\alpha\beta}$ matrix elements.
Then, use Eq. (7.32) of the book to express the dual tensor in terms of $\vec{e}$ and $\vec{b}$. Finally, compare ${}^*E^{\alpha\beta}{}_{,\alpha} = 0$ with the homogeneous Maxwell equations, Eq. (7.26) and (7.27) of the book.

Prepare for the Midterm on W October 17!

Set 8:

(20) Homework, due M 10/22/2017 before class. Distance in a saddle.

Consider a saddle described by the function

$$z = f(x, y) = x^2 - y^2$$

in arbitrary units. A two-dimensional being walks a distance $r = 10$ from the origin $x = y = 0$ of the saddle into its $\hat{x}$ direction. At which value of x is it? Give at least three significant digits.

Set 9:

(21) Homework, due M 10/29/2018 before class. Radar distance.

The radar distance is defined as $(c/2)\,\triangle\tau$ with $\triangle\tau$ being the proper time elapsed on a standard clock between emission and reception of a radio echo.

Observers at two fixed points A and B in a stationary gravitational field, respectively at Φ_A and Φ_B, determine the radar distance between them by use of standard clocks. Let L_A and L_B be the determinations made at A and B respectively. Find L_A/L_B.

(22) Homework, due M 10/29/2017 before class. Rindler 9.8.

Hints: First, complete the square for dt. Then, apply the transformation $t' = t + f(x, y)$ (called a "gauge transformation" in the context of gravity) and find a function $f(x, y)$ so that the metric becomes static (i.e., has no cross terms $dx\,dt$ and so on).

(23) Classwork, due M 10/29/2018 in class. Partial Derivatives.

To limit the scope of this classwork, take only positive roots. Let $x' = x^2\,y^2$ and $y' = x^2/y^2$.

(a) Find $x(x', y')$ and $y(x', y')$.
(b) Calculate

$$\frac{\partial}{\partial x'}\,\frac{\partial x'}{\partial x} \quad \text{and} \quad \frac{\partial}{\partial x}\,\frac{\partial x'}{\partial x'}\ .$$

Express the result in terms of x and y.

(c) Compare the previous results with

$$\frac{\partial^2 x'}{\partial x\, \partial x}\, \frac{\partial x}{\partial x'} + \frac{\partial^2 x'}{\partial x\, \partial y}\, \frac{\partial y}{\partial x'}$$

in terms of x and y. List the result for each of the two contributions separately before adding them up.

(d) Explain

$$\frac{\partial}{\partial x}\, \frac{\partial}{\partial y} = \frac{\partial}{\partial y}\, \frac{\partial}{\partial x} \quad \text{versus} \quad \frac{\partial}{\partial x'}\, \frac{\partial}{\partial x} \neq \frac{\partial}{\partial x}\, \frac{\partial}{\partial x'}\ .$$

(e) Consider 2D coordinates systems which are related by

$$x^{1'} = (x^1)^2\, (x^2)^2 \quad \text{and} \quad x^{2'} = (x^1)^2/(x^2)^2\ .$$

Calculate $p^{1'}_{1i}\, p^{i}_{1'}$ in terms of x^1 and x^2. The p (partial derivatives) symbols are defined in chapter 7 of Rindler, p.132.

Set 10:

(24) Homework, due F 11/2/2018 before class. Consider a flat 2D plane mapped by the standard polar coordinates.

(a) Write down ds^2.
(b) What are the coefficients g_{ij}?
(c) What are the coefficients g^{ij}?
(d) Calculate the Christoffel symbols (affine connections) of second kind Γ^i_{jk} for this system.
(e) Using

$$\ddot{x}^i + \Gamma^i_{jk} \dot{x}^j\, \dot{x}^k = 0$$

derive the two coupled, second-order differential equations that describe the motion of a free particle in this system.

(25) Homework, due F 11/2/2018 before class. Verify that the differential equations

$$\ddot{r} = r\, \dot{\theta}^2 \quad \text{and} \quad \ddot{\theta} = -\frac{2}{r}\, \dot{r}\, \dot{\theta}\,, \quad \text{where} \quad \cdot = \frac{d}{ds}\,,$$

are fulfilled by

$$r = \sqrt{y_0^2 + s^2} \quad \text{and} \quad \tan(\theta) = \frac{y_0}{s}\,.$$

Here y_0 is a constant. Describe the motion in the x-y plane.

Find at least one more solution of these differential equations.

Set 11:

(26) Homework, due W 11/14/2018 before class. Consider the space-time metric $d\vec{s}^2 = dt^2 - dl^2$, where dl^2 is an arbitrary and possibly time-dependent metric. Are coordinate lines with t variable and the other coordinates fixed at constant values geodesic? Hint: Investigate whether the geodesic equations of the Lagrangian

$$L = \dot{t}^2 - g_{ij}\,\dot{x}^i\,\dot{x}^j$$

allows for the proposed solutions.

(27) Homework, due W 11/14/2018 before class. Exercise 10.6 of Rindler.

(28) Homework, due W 11/14/2018 before class. Exercise 14.1 of Rindler.

Set 12:

(29) Homework, due F 13/14/2018 before class. Exercise 14.2 of Rindler.

(30) Homework, due F 13/14/2018 before class. Use Gaussian units ($g\,cm\,s$) and the following values for this problem:
Speed of light: $3 \times 10^{10}\,[cm/s]$.
Newton's gravitational constant: $G = 6.674 \times 10^{-8}\,[cm^3/(s^2\,g)]$.
Radius of the visible universe: $r_u = 4.4 \times 10^{28}\,[cm]$.
Ordinary matter in the visible universe: $10^{56}\,[g]$.
The cosmological constant Λ leads to a modified Poisson equation in the Newtonian limit (Rindler, p.304). The observed value (Planck satellite in 2015) is

$$\Lambda = 1.11 \times 10^{-56}\,[1/cm^2]\,.$$

Compare the absolute values of the implied negative energy density (sometimes called "dark energy") with the energy density due to ordinary matter. State your answer in percent of the two absolute values added up.

(31) Classwork, due F 11/16/2018 in class. Schwarzschild metric.
For the diagonal metric

$$d\vec{s}^{\,2} = a\,(dx^1)^2 + b\,(dx^2)^2 + c\,(dx^3)^2 + d\,(dx^4)^2\,,$$

where a, b, c, d are functions of the coordinates, the R_{11} compoment of the Ricci tensor is given by

$$\begin{aligned} R_{11} &= +\,\beta\,a_{22} + \gamma\,a_{33} + \delta\,a_{44} \\ &\quad +\,\beta\,b_{11} + \gamma\,c_{11} + \delta\,d_{11} \\ &\quad -\,\beta^2\,b_1^2 - \gamma^2\,c_1^2 - \delta^2\,d_1^2 \\ &\quad -\alpha\,a_1\,(\,+\,\beta\,b_1 + \gamma\,c_1 + \delta\,d_1\,) \\ &\quad -\beta\,a_2\,(\,\alpha\,a_2 + \beta\,b_2 - \gamma\,c_2 - \delta\,d_2\,) \\ &\quad -\gamma\,a_3\,(\,\alpha\,a_3 - \beta\,b_3 + \gamma\,c_3 - \delta\,d_3\,) \\ &\quad -\delta\,a_4\,(\,\alpha\,a_4 - \beta\,b_4 - \gamma\,c_4 + \delta\,d_4\,) \end{aligned}$$

where we use the notation (Rindler Appendix)

$$\alpha = \frac{1}{2a}\,, \quad \beta = \frac{1}{2b}\,, \quad \gamma = \frac{1}{2c}\,, \quad \delta = \frac{1}{2d}\,,$$

and for $i, j = 1, 2, 3, 4$.

$$a_i = \frac{\partial a}{\partial x^i}\,, \qquad a_{ij} = \frac{\partial^2 a}{\partial x^i\,\partial x^j}\,, \quad \text{and so on.}$$

We want to construct a stationary, static, spherically symmetric metric of the form

$$d\vec{s}^{\,2} = e^{A(r)}\,dt^2 - e^{B(r)}\,dr^2 - r^2\,(d\theta^2 + \sin^2\theta\,d\phi^2)\,,$$

where use units with speed of light = 1.

(a) Let $r = x^1$, $\theta = x^2$, $\phi = x^3$ and $t = x^4$ and identify the non-zero contributions to $R_{11} = R_{rr}$.

(b) Use the result of (a) to proof equation (11.3) of the book

$$R_{rr} = \frac{A_{11}}{2} - \frac{A_1\,B_1}{4} + \frac{(A_1)^2}{4} - \frac{B_1}{r}\,.$$

Set 13:

(32) Homework, due M 11/19/2018 before class. Use the appendix of Rindler, p.419/420, or better the version posted on the Web, to show Eq. (11.4) for the Ricci tensor element R_{tt}.

Set 14:

(33) Homework, due W 11/28/2018 before class. Gravitational Units.

In the following use $G = 6.674 \times 10^{-11}\,[m^3\,kg^{-1}\,s^{-2}]$ for the gravitational constant, $m_e = 5.972 \times 10^{24}\,[kg]$ for the mass of the earth, which is considered to be a perfect sphere of radius $r_e = 6371 \times 10^3\,[m]$ of uniform mass density. Perform calculations for this problem to the three leading digits (rounding the fourth digit).

(a) Calculate the Scharzschild radius r_e^s of the earth in units of meter $[m]$ and, subsequently, the ratio r_e^s/r_e.
Continue using gravitational units defined by $c = G = 1$.

(b) Write down the values for m_e and r_e in units of seconds [s]. Subsequently, calculate the ratio $2\,m_e/r_e$.

(c) Write down the values for m_e and r_e in units of years [y]. Subsequently, calculate the ratio $2\,m_e/r_e$.

(d) Write down the values for m_e and r_e in units of meters [m]. Subsequently, calculate the ratio $2\,m_e/r_e$.

(e) Write down the values for m_e and r_e in units of light years [ly]. Subsequently, calculate the ratio $2\,m_e/r_e$.

(34) Homework, due W 11/28/2018 before class. Solar system: Schwarzschild radius, ruler distance and radar distance.

In the following use $G = 6.674 \times 10^{-11}\,[m^3\,kg^{-1}\,s^{-2}]$ for the gravitational constant, $m_s = 1.9886 \times 10^{30}\,[kg]$ for the mass of the sun, which is considered to be a perfect sphere of radius $r_s = 6957 \times 10^5\,[m]$ of uniform mass density. Assume $r_1 = 149.6 \times 10^9\,[m]$ for the coordinate distance of the earth to the center of the sun. Neglect the mass of the earth (or assume it to be replaced by a satellite). Perform calculations for this problem to the three leading digits (rounding the fourth digit).

(a) Calculate the Scharzschild radius r_s^s of the sun and, subsequently, the ratio r_s^s/r_s.

(b) Calculate the ruler (geodesic) distance from the earth to the surface of the sun and compare it with the corresponding coordinate distance.

(c) Calculate the radar distance from the earth to the surface of the sun and compare it with the corresponding coordinate and ruler distances.

(35) Homework, due W 11/28/2018 before class. Isotropic form of the Schwarzschild metric. Substitute

$$r = \left(1 + \frac{m}{2\overline{r}}\right)^2 \overline{r}$$

into the Scharzschild metric to derive its "isotropic" form

$$d\vec{s}^{\,2} = \frac{(1 - m/2\overline{r})^2}{(1 + m/2\overline{r})^2}\, dt^2 - \left(1 + \frac{m}{2\overline{r}}\right)^4 \left(d\overline{x}^2 + d\overline{y}^2 + d\overline{z}^2\right) .$$

Set 15:

(36) Homework, due W 12/5/2018 before class. Shapiro time delay.

(a) Find the Shapiro time delay in the approximation of Rindler p.237 for X_1 the distance from Earth to Sun, X_2 the distance from Mercury to Sun and R the radius of the Sun. State the result in seconds. You may take the parameter values from Wikipedia. Calculate to at least three significant digits.

(b) Repeat the above estimates with Mercury replaced by Venus.

(37) Homework, due W 12/5/2018 before class. Mercury precession.

(a) Estimate the Einsteinian advance of the perihelion of Mercury in radians from the equation $\triangle \approx 6\pi\, m^2/h^2$, where you may use the approximation $\omega = \sqrt{m/r^3}$ to find the specific angular momentum of Mercury from its frequency.

(b) Find from the previous result the precession of Mercury over 100 earth years and express the result in arc seconds.

(38) Homework, due M 12/5/2018 before class. Radial coordinate velocity of light in Schwarzschild spacetime. Exercise 11.2 of Rindler.

Prepare for the Final

Thursday, December 13, 2018, 10 am - 12 pm.